Standardized Square Footings

OF

Reinforced Concrete

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By

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INTRODUCTION.

The text of this booklet is merely the explanation of the use of the diagrams, devised by the writer for the solution of the problems arising in connection with the design and review of Square Footings of Reinforced Concrete.

The economic advantages offered by concrete footings reinforced with steel bars, as contrasted with the earlier form, using structural steel shapes, are so great, that the importance of correct design is increased in the ratio of their use. Undertaken by the writer for the purpose of standardizing work in our own office, the task has developed to a point where it seems apparent that the results will be useful to many designers, and are, therefore, made available by publication.

There has also been given a brief summary of the best comparative information available to the writer with regard to methods in general use by designers, and the results of experimental determination of stresses by test, for the purpose of giving anyone using the diagrams a correct sense of the meaning of the results obtained with reference to other methods. Also comparative information is given as to the factors of safety, which exist in connection with the use of working stresses, as defined by Ordinances and other authorities, in contrast with ultimate values obtained by experiment.

The need of such general information is amply shown in the data given with regard to bending moments.

Primary assumptions used quite generally by practical designers are shown to vary as much as one hundred per cent. and give results at variance with each other to the extent of fifty per cent.

The range of opinion and practice with reference to the calculation of reinforcement for diagonal tension is in even a more chaotic state.

The principal diagrams are presented as two-line nomographs, which, in the form used, furnish a solution of what would be a series of lines on the ordinary graph, by the use of only two lines, thus reducing the objectionable features, caused by the complexity of numerous lines on the ordinary graph, to a minimum.

These diagrams are to be considered as a special form of slide-rule devised for the particular purpose in question, which permit the solution of problems by the use of standard formulae in any way the user may see fit, only governed by the requirements of local Ordinances.

Most of the formulae are in the form used by Professor Arthur N. Talbot of the University of Illinois. This is, of course, due to the circumstance that the only tests on column footings that have ever been made are those described in Bulletin 67 of the University of

Illinois, carried out under his direction. It, therefore, follows that the notation used by him will be best adapted to the purpose in hand, and where in a few cases it is departed from, both notations are given. Where other formulae are required, general in character, those recommended by the Joint Committee* have been used.

The diagrams are entirely general in nature, and care has been used to prevent the intrusion of anything not warranted by competent authority. They may consequently be considered as tools, which will enable their user to avoid the laborious arithmetical computation connected with the use of footing formulae, in exactly the same manner that the slide-rule makes it possible for one to use mathematical expressions with greater facility and saving of time than is possible without it. While a slide-rule may be considered an adjustable nomograph, the specialized nomograph has a further advantage not possessed by the ordinary slide-rule, of eliminating to a very great extent arithmetical error in the results of any given problem. Since the multiplication of constants has been effected and recorded in the form of a smooth curve, the chance of error in any problem is minimized in direct proportion to the number of operations eliminated, in the performance of the work.

It will be seen that it in no way supersedes an understanding of the formulae themselves any more than would be the case with the slide-rule, but it does tremendously enhance one's understanding of the subject by increasing the rapidity with which he gains an intelligent conception of the effect upon his results of any given variation in the factors. For this reason it will be found useful, not only to the expert, who does not need the simple facts presented for the benefit of the less sophisticated designer (but whose facility of action will be increased by the use of a better tool), but it also furnishes the beginner with reliable methods which make it possible for architects and engineers entrusting design to subordinates to save much wasted time in securing results that are correct and satisfactory. It also gives to them the power to check results here and there without loss of their own time.

It likewise affords valuable assistance for rapid analysis to the examining authorities of Building Departments, and to students and teachers, for purposes of instruction or the working up of experimental data.

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*The latest report of the Joint Committee on Reinforced Concrete appears in the Proceedings of the American Society of Civil Engineers for February, 1913.

The Committee carries representation and authority from the American Society of Civil Engineers, the American Society for Testing Materials, American Railway Engineers and Maintenance of Way Association, and the Association of American Portland Cement Manufacturers. Its recommendations are the careful findings of experts best qualified in their several lines to establish limits, and its report unquestionably represents the best authority extant on the subject.

For the sake of uniformity of practice, everyone desiring to promote rational design will support its conclusions until they are changed by further experience or replaced by better authority.

The nomographs have been printed on cloth, on a larger scale, separate from the book itself, so as to be available on the draughting board, and after having acquainted oneself with the contents of the booklet, it will only be needed for reference, and need not incumber one's working space.

The writer desires to use this opportunity to express his sense of obligation to Professor Talbot for the thoroughness with which the data contained in Bulletin 67 has been presented, and in this connection to say that the results herein given are original only in their presentation of methods for using the information made available to designers by these important experiments. Whatever value attaches to them is merely that they furnish a practical interpretation of the work of Professor Talbot.

That the matter herein presented may prove useful to designers, may insure more rational footings, thus tending to unify practice, and may render effective assistance in promoting the use of information provided to the profession by Professor Talbot in Bulletin 67, is the purpose of its publication.

L. A. H.

San Francisco, Calif.
May, 1915.

NOTATION.

Square Concrete Footings.

$^{\Delta}$ used as a superscript indicates that value of symbol affected is to be found in the diagrams.

	DIAGRAMS.	TALBOT.
	For dimension symbols see drawing of typical footing on page 7.	
w	Allowable soil reaction, pounds per sq. ft.	
W	Load to soil from column and approximate weight of footing.	
E	One side of square concrete column, shoe, C. I. base or steel slab.	
L	One side of square footing.	l
E/L	Ratio between widths of footing and concrete column or other load transmitting element.	
B	One side of square cap.	a
B/L	Ratio between widths of cap and footing.	
$\frac{L\text{-}B}{2}$	Projection of footing beyond cap.	c
H	Combined depth of footing and cap above steel.	
j	Distance from center of steel to center of compression.	
d	Depth of footing from upper face to center of steel.	
d/L	Ratio of depth to width of footing.	
M	Bending moment, or equivalent width of beam in which steel for bending moment is distributed.	b
m	No. of bars used for bending moment, and to calculate Bond Stress.	

M_s Resisting moment of steel in width M.
M_c Resisting moment of concrete in width M.
A_s Area of steel in width M—Sq. ins.
O Width beyond M on either side at corners of footing.
As_1 Area of steel used in width O.
o Perimeter of one reinforcing bar.
Av_s Area of steel used for stirrups, as reinforcement against diagonal tension.
N Width within which any bent bars used for reinforcement against diagonal tension are located.

DIAGRAMS.

Av_b Area of steel used for bent bars in width N.

V_p External Shear in width E for depth H, or in width B for depth d on one side of footing only.

v_p Allowable unit resistance to shear at same points.

V_n External shear at any point S_1 selected as place of measurement for diagonal tension, other than E, B, or S.

v_n Unit resistance to shear, caused by V_n.

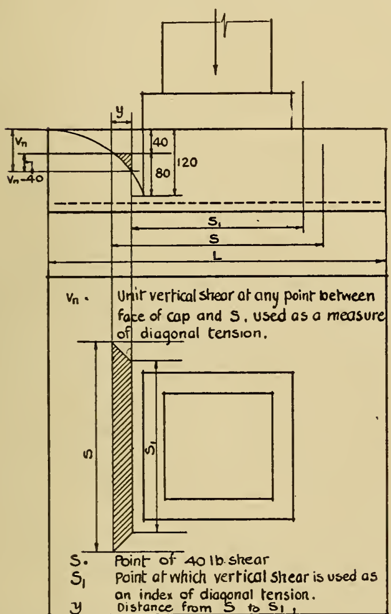


FIG. 1. ILLUSTRATING SYMBOLS USED IN DIAGONAL TENSION FORMULAE.

S/L Ratio between length S (= one side of square where unit shear equals 40 lbs.) and size of footing L.

S_1/L Ratio between length S_1 (= one side of square at point selected as place of measurement for diagonal tension) and size of footing L.

y/L Ratio between distance y (distance between point selected at which to measure shear as an index of diagonal tension and point of allowable 40 lb. shear in concrete) and size of footing =

$$\frac{\frac{S/L}{L} - \frac{S_1/L}{L}}{2}$$

u_a Average unit bond stress.

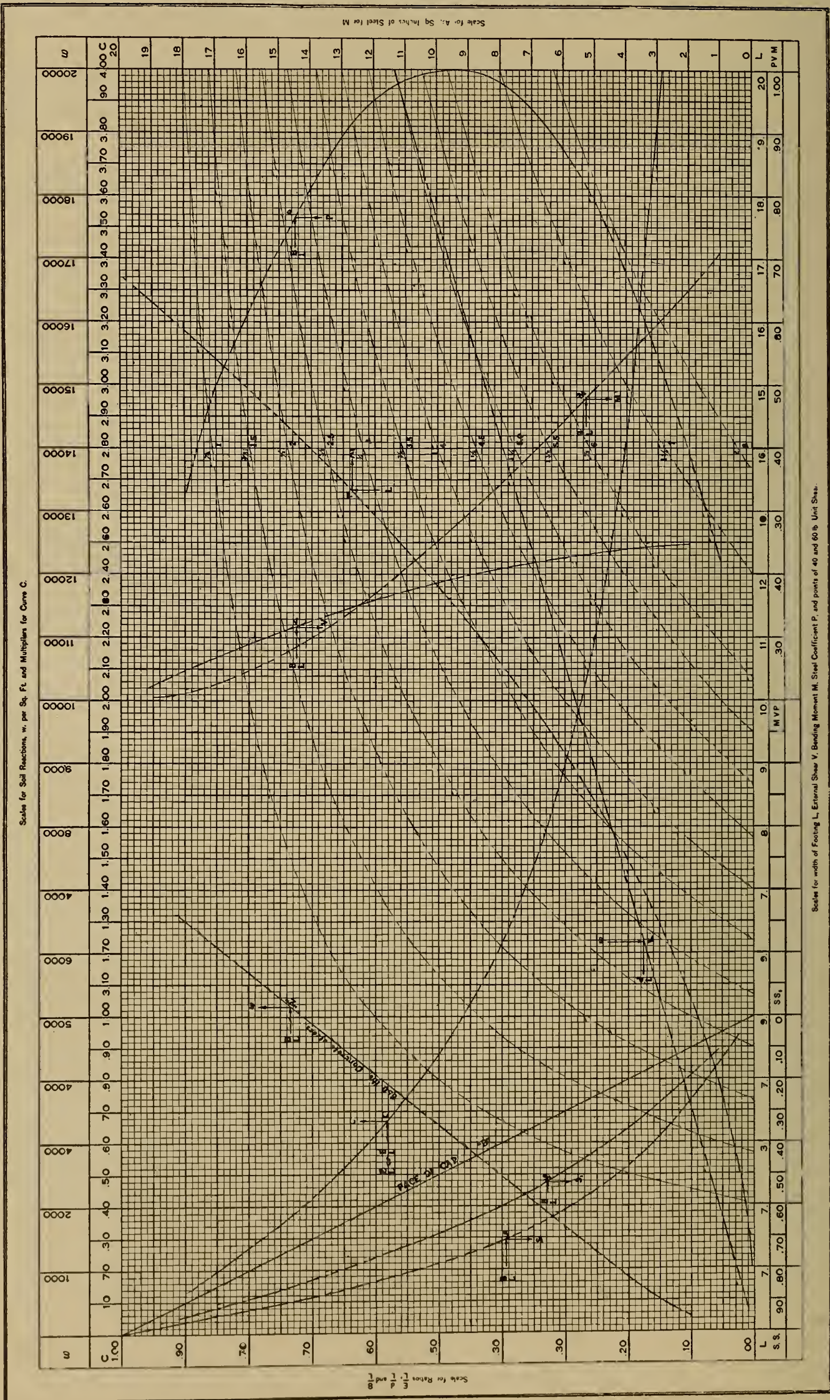
f_s Allowable unit stress in steel.

f_c Allowable unit stress in concrete.

k Distance from Neutral Axis to top of footing.

Other letters as shown on drawing of typical footing, page 7.

NOMOGRAPHS FOR STRESSES



DESIGN DATA

*Superscript $^{\Delta}$ indicates values to be found in diagrams.

	BASIS OF DESIGN (1) Constant maximum unit shear "v" = 120 lbs.	(2) 650 lbs. in Concrete	FOR ANALYSIS ONLY (3) Any maximum shear less than 120 lbs.
L	$\sqrt{\frac{W}{w}}$ — w = Soil load per sq. ft. W = Column Load = wL^2	Same as (1)	Same as (1)
E	Width of concrete column, C. I. base, or steel slab	do	do.
H	C K L. C^{Δ} for $\frac{E}{L}$ used and K^{Δ} for soil load assumed*	$I = H - d_c$	do. $I = H - d_n$
B	Judgment of designer to provide proper shear and bending	Same as (1)	do.
	conditions in cap. Minimum projection beyond "E" usually 6"	do	d (1) is min. allowable shear depth ∴
d	C K L. C^{Δ} for $\frac{B}{L}$ used and K^{Δ} for soil load assumed	$d_c^2 = \frac{2M}{650\,Kjb}$	Any other depth d_n must be greater
D	d + 3," or ordinance requirements		
M	$B + 2d + \left(\frac{L-(B+2d)}{2}\right)$	Same as (1)	Same as (1)
A_s	PA for width M. P^{Δ} for $\frac{B}{L}$ used A^{Δ} for "L" of footings	$\left(\frac{d}{d_c}\right)A_s$	$\left(\frac{d}{d_n}\right)A_s$
O	$\frac{L-M}{2}$	Same as (1)	
A_{s_1}	For width O. $\left(\frac{1}{2}\right)\left(\frac{O}{L}\right)A_s$	$\left(\frac{d}{d_c}\right)A_{s_1}$	$\left(\frac{d}{d_n}\right)A_{s_1}$
A_{v_s}	Stirrups $\left(\frac{v}{2}\right)\left(\frac{\frac{S}{L}+\frac{S_1}{L}}{2}\right)$ (y) (144) L^2/16000. s^{Δ} and s_1^{Δ} for $\frac{B}{L}$ used	Same as (3)	$\left(\frac{v_n}{120}\right)A_{v_s}$
N	B + 2x. Point where bend in bars occurs	Same as (1)	Same as (1)
A_{v_b}	Bent Bars. 0.7 A_{v_s}	0.7 $\left(\frac{v_n}{120}\right)A_{v_s}$	Same as (2)
f_c	Negligible below line Z. Above the line see Heading (2)	See d_c above	Same as (1)
u_s	Use limit bond line above intersection of $\frac{B}{L}$ and L for straight bars. See text for bent bars.		
I	H — d		
F	$\frac{B-E}{2}$ *		

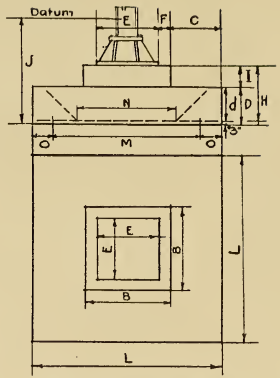
BAR DATA

o = Perimeter a = Area

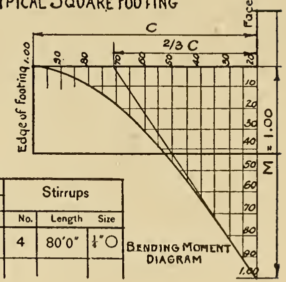
Size	SQUARE o	SQUARE a	SQUARE Tens.	ROUND o	ROUND a	ROUND Tens.
1/4	1	.06	1000	.79	.05	800
3/8	1½	.14	2250	1.18	.11	1760
1/2	2	.25	4000	1.57	.196	3140
5/8	2½	.39	6250	1.96	.31	4950
3/4	3	.56	9000	2.36	.44	7050
7/8	3½	.77	12300	2.75	.60	9600
1	4	1.00	16000	3.14	.79	12600
1⅛	4½	1.27	20200	3.54	.99	15800
1¼	5	1.56	25000	3.94	1.23	19700
1⅜	5½	1.89	30300	4.33	1.48	23700
1½	6	2.25	36000	4.72	1.77	28400
1¾	7	3.06	49000	5.50	2.41	38600
2	8	4.00	64000	6.28	3.14	50000

GRIP LENGTH

DESCRIPTION	WORKING STRESS	GRIP
Def. Bars	100 lbs.	40 diam.
Plain Bars	80 lbs.	50 diam.



TYPICAL SQUARE FOOTING



BENDING MOMENT DIAGRAM

SCHEDULE FOR TYPICAL SQUARE FOOTINGS

NOTES:* Caps must be run monolithic with Footings.
Extend Splice Bars, equal to Column area for reinforced concrete columns, well into footing depth.
All bars except continuous stirrups must be of deformed type.
All bent bars to have anchor hooks bent through 180° of 5 bar diameters and short tangent.
Clearance of not less than 2" between all steel and adjacent concrete surface.

Distribute Tension Bars in width N so that a bent bar is located at either end of N and the remaining bars spaced uniformly between with bent bars alternating when possible.

Column Numbers	No.	Soil load 7000 Column Loads	B	D	E	F	H	J	L	M	N	O	Tension Bars, One Way, Equal spacing width M: No.	Length	Size	Tension Bars, One Way, Equal spacing width O: No.	Length	Size	Bent Bars, One Way, Equal spacing width N: No.	Length	Size	Stirrups: No.	Length	Size
34, 36, 42, 64, 67, 96	6	700000	4'-6"	2'-3¼"	3'-6"	6"	2'-11"	4'-8"	10'-0"	9'4"	5'4"	4"	18	9'-8"	⅜"□	1	9'-8"	½"□	4	11'10"	⅜"□	4	80'0"	¼"O

* To be used or not as designer decides

Designation of Lines on Diagrams.

All diagram problems are solved by following line marking desired point of use, on scale of origin, horizontally or vertically as case is, to its intersection with the curve relating to the problem; thence at right angles to first direction to scale relating to answer.

Arrows at point, where designating letters are placed on curves, indicate direction of movement from scale of origin to scale for answer.

Scales are referred to as "Right Scale," meaning scale on right hand of chart; "Left Scale for B/L"—E/L or d/L, all using the same scale; "Upper Scale for w" and "Lower Scale for L," etc.

Depth.

K Factor for variation in soil pressure w.
C Factor for variation of cap width B/L.

Area of Steel.

P Factor for variation of cap width B/L.
A Factor for variation of footing width L.

Diagonal Tension.

S Point of 40 lb. unit shear with reference to 120 lbs. at face of cap, for variation of B/L.

S_1 Point of 60 lbs. unit shear with reference to 120 lbs. at face of cap, for variation of B/L, or any point between S and face of cap.

B Face of cap for any B/L.
Intercepts between lines S and S_1 on line of B/L used give value of y for one value of v_n. For any value of v_n Intercept y may be obtained from diagram in manner explained elsewhere. Intercept on line of B/L used, between lines B and S, give distance of point of 40 lb. shear from face of cap for use in fixing position of web reinforcement.

Bond.

Curves of equal bond stress = 100 lbs. designated by the size of the bar, have been placed inconspicuously on the chart so that size of square deformed bars is determined at a glance.

Also methods of using the diagram curves in connection with table of bar data to adapt results to deformed round bars, plain square or round bars, and to establish the amount of tension steel that can be bent up within maximum bond limits, are given in explanations. (See Page 21)

Limit Stress in Concrete.

Z Where values of soil pressure w and cap width B/L inter-

sect above this line, footing depth will be governed by limit stress of 650 lbs. in concrete. For intersections of these factors below this line, the concrete stress will be less than 650 lbs. and need not be calculated.

External Shear.

V While not required for standardized design, examination and analysis may sometimes make it convenient to know the External Shear at any point in a footing. This is given by the expression

$$V = V^{\Delta} W$$

Bending Moment.

M While not required for direct calculations, since it is integrated into the curves for determination of Steel Area, it becomes necessary when depth is determined by stress in concrete and is convenient for purposes of analysis. The desired value is expressed

$$M = (M^{\Delta}) \frac{WL}{10}$$

Working Stresses and Diagram Constants.

Allowable Unit Working Stresses.

Tension in steel	16000 per sq. inch
Shear in steel	11200 " " "
Max. Fiber Stress in Concrete	650 " " "
Max. Vertical Shear in plain concrete	40 " " "
Max. Vertical Shear in reinforced concrete with diagonal tension provided for by reinforcement	120 " " "
Bond Stress—	
Plain Bars	80 " " "
Deformed Bars	100 " " "
Bearing on Concrete	500 " " "

Value of Constants.

j, .875. *k*, .375.

Where it is desirable for purpose of analysis or experiment to use more accurate values of these constants, the diagram results containing these factors may be multiplied by the constant used and properly affected by the more correct value.

FORMULAE.

WORKING TRANSFORMATION.	REFERENCE.
(1) $L = \sqrt{\frac{W}{w}}$	SIZE OF FOOTING.
	PUNCHING SHEAR.
(2) $d = C^{\triangle} K^{\triangle} L$	$d = \frac{(L^2 - B^2)w}{(4Bj)(120)} = \frac{V}{4Bjv}$
	AREA OF ONE WAY TENSION STEEL FOR WIDTH *M*.
(3) $A_s = P^{\triangle} A^{\triangle}$	$A_s = \frac{(\frac{1}{2}ac^2 + .6\ c^3)w}{(16000)(7/8)(d)}$
	$M = B + 2d + \frac{L - (B + 2d)}{2}$
	AREA OF ONE WAY TENSION STEEL FOR WIDTH *O*.
(4) $A_{s1} = (\frac{3}{4})(\frac{O}{L})A_s$ *Fractional coefficient ¾ may be replaced with any value less than unity, dictated by the judgment of the designer.*	$O = \frac{L - M}{2}$
	VERTICAL SHEAR AS A MEASURE OF DIAGONAL TENSION AT ANY POINT S_1 GOVERNED BY ORDINANCES OR AUTHORITY.
(5) $V_n = V^{\triangle} W$	EXTERNAL SHEAR. $V = (ac + c^2)w$
(6) $v_n = \frac{V^{\triangle} W}{S_1\ jd}$	UNIT SHEAR. $v = \frac{V}{bjd}$
	AREA OF STEEL FOR DIAGONAL TENSION. *General.*
(7) $A_{vs} = \frac{144 \left(\frac{v_n - 40}{2}\right)\left(\frac{\frac{S}{L} + \frac{S_1}{L}}{2}\right)(y)(L^2)}{16000}$	$A_{vs} = \frac{V\ s}{f_s\ jd}$

If $v_n - 40$ is less than 40 lbs., no diagonal tension, beyond the allowable stress taken by the concrete, exists at the point selected. If some minimum amount is required and used, as for instance at the point S_1, where $v_n = 60$, formula may be written—

MINIMUM.

$$A_{vs} = \frac{144\left(\frac{60-40}{2}\right)\left(\frac{\frac{S}{L}+\frac{S1}{L}}{2}\right)(.05)(L^2)}{16000}, \text{ or}$$

(8) $$A_{vs} = .0023\left(\frac{S}{L}+\frac{S1}{L}\right)L^2, \text{ or } \frac{\left(\frac{S}{L}+\frac{S1}{L}\right)L^2}{444}$$

For V_n 120 the formula may be expressed

MAXIMUM.

$$\frac{144\left(\frac{120-40}{2}\right)\left(\frac{\frac{S}{L}+\frac{S1}{L}}{2}\right)(.15)(L^2)}{16000} \text{ or}$$

(9) $$A_{vs} = .027\left(\frac{S}{L}+\frac{S1}{L}\right)(L^2)$$

(10) $Av_b = 0.7\,Av_s$ Bent bars at 45°.

AVERAGE BOND STRESS, FOR ONE BAR. TENSION VALUES @ 16,000 LBS. PER SQUARE INCH.

SQUARE BARS.

(11) $$\text{Tension} = (1000)(0^2) = 1ou$$

ROUND BARS.

(12) $$\text{Tension} = (1273)(0^2) = 1ou$$

EMBEDMENT FOR 45° BENT BARS, FROM LINE OF DIAGONAL TENSION RUPTURE TO UPPER CLEARANCE BEND.

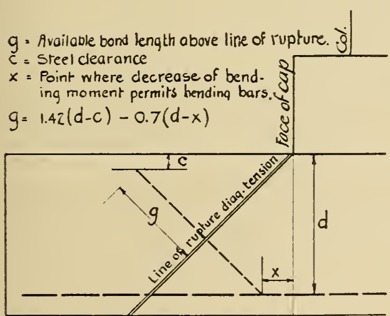


FIG. 2. ILLUSTRATING FORMULA 13. GRIP FOR BENT BARS.

Let available grip length . . . . $= g$.
Clearance . . . . $= c$.
Distance of bend from face of cap $= x$.

(13) Then $g = 1.42\ (d - c) - .7\ (d - x)$.

MAXIMUM BOND STRESS.

(14) $$u = \frac{V}{m o j d}$$

CONCRETE STRESS TO DETERMINE DEPTH.

(15) $$d = \frac{2M}{650\ kjb}$$

Use of Diagrams for Design.

A general problem is assumed, and the explanations for the use of curves is followed by specific application to the particular problem.

The problem to be solved is as follows, and reference is made to the typical footing on Plate 2, page 7, for symbols used:

Design a square footing for a plate and angle column resting on a cast iron base, 3′ 6″ square. The load delivered to the soil by the dead and live load carried by the column inclusive of the approximate weight of the footing itself is 700,000 lbs.

Allowable soil pressure has been determined as 7,000 lbs. per square foot.

Width. $$\text{Then } L = \sqrt{\frac{700{,}000}{7{,}000}} = 10'\ 0''$$

Depths. To ascertain the depth required for 120 lbs. shear through the cap and footing at the edge of C. I. base, find ratio between width of C. I. base and one side of footing, or

$$E/L = \frac{3.5}{10} = .35$$

Opposite .35 on Left Scale, find point on Curve C, vertically below 1.48, the answer, on Upper Scale for C.

Below 7000 on Upper Scale of soil pressures w, find point on Curve K opposite .199, the answer, on Left Scale.

Then $H = C^{\Delta}\ K^{\Delta}\ L = (1.48)\ (.199)\ (10'\ 0'') = 2.95$ ft.

Results may be recorded in typical footing schedule, as on page 7.

To ascertain the depth required for 120 lbs. shear through the footing alone around edge of cap, find ratio between width of cap and footing, or

$$B/L = \frac{4.5}{10} = .45$$

Opposite .45 on Left Scale, find point on Curve C vertically below 1.04, the answer, on Upper Scale for C.

The value of K for 7000 lbs. soil pressure has already been determined, as .199.

Then $d = C^{\Delta} K^{\Delta} L = (1.04)(.199)(10' \ 0'') = 2.07$ ft.

$D = d + 3 = 2.32$ ft.

$I = H - d = 2.95 - 2.07 = 0.88$ ft.

Area of Steel.

Opposite B/L for width of cap used, viz., .45, on Left Scale, find point on Curve P vertically above .999, the answer on Lower Scale for P.

Above width of footing 10, on Lower Scale L, find point on Curve A opposite value 7.15, the answer on Right Scale.

Then $A_s = P^{\Delta} A^{\Delta} = (.999)(7.15) = 7.13$ sq. ins.

Size of bars for tension steel will be decided upon when bond conditions are considered.

Width of Footing in Which A_s is to be Uniformly Distributed.

$$M = B + 2d + \frac{L - B + 2d}{2}$$

$$4.5 + 4.14 + \frac{10 - 8.64}{2} = 9.32 \text{ Ft.}$$

$$O = \frac{L - M}{2} = \frac{10 - 9.32}{2} = 0.34 \text{ Ft.}$$

Area of Steel for Width O.

$$A_{s1} = \left(\tfrac{3}{4}\right)\left(\frac{O}{L}\right) A_s$$

$$= \left(\tfrac{3}{4}\right)\frac{0.34}{10}(7.13) = 0.18 \text{ sq.ins.}$$

The factor ¾ may be changed by the designer to any value less than unity, which in his judgment is warranted.

Diagonal Tension. The point selected at which to measure vertical shear is $1d$ from face of cap $= 2.07$ ft.

Opposite B/L used for cap width viz. .45, on Left Scale, find point on Curve S vertically above value .745, the answer on Lower Scale S.

Then $S/L = .745$ or $S = 7.45$.

S represents one side of square, where the resistance to shear equals 40 lbs. The length of one side of square at point selected for diagonal tension index will be

$$S_1 = 4.50 + 2.07 + 2.07 = 8.64 \text{ ft.}$$

As S_1 is greater than S, the vertical shear at point selected will be less than 40 lbs. and need not be calculated.

As Joint Committee report of 1913 allows 120 lbs. vertical shear "when tension normal to shearing plane is provided for by steel," some minimum amount should be used, and it is proposed to make the calculations for 20 lbs. of shear in excess of the 40 lbs. allowed in the concrete to illustrate the use of diagrams, leaving to the designer's judgment and the requirements of local codes the decision as to what is allowable in practice.

Using Formula (7),

S/L as found above $= .745$.

Opposite B/L used for cap width, viz. .45, on Left Scale, find point on Curve S_1 vertically above value .65, the answer on Lower Scale S.

On the line of $B/L = .45$, the intercept distance between Curves S_1 and S may be counted directly in unit subdivisions, and is found to be 4.9 units, or $y = .049$, or may be figured as—

$$\frac{S/L - S_1/L}{2} = \frac{.745 - .65}{2} = .0475$$

Then using Formula 7,

$$A_{v_s} = \frac{144 \left(\frac{60 - 40}{2}\right)\left(\frac{.65 + .745}{2}\right) (.0475)\ (10^2)}{16000} \text{ or}$$

$$\frac{(144)\ (10)\ (.6975)\ (.0475)\ (10)\ (10)}{16000} = 0.3 \text{ sq. ins.}$$

or about 5% of area of tension steel for M.

If bent bars are used, the area required will then be

$$Av_b = (.07)\,(Av_s) = (0.7)\,(0.3) = 0.21 \text{ sq. ins.}$$

or about 3% of steel required for tension in width M.

Where the point selected for an index to diagonal tension is such that the total shear is 60 lbs., as in this case, the shorter formula (8) may be used, viz.:

$$A_{vs} = .0023 \quad (S + S_1) \quad L^2$$

$$= (.0023)(.65 + .745)(100) = .32 \text{ sq.ins.}$$

Assume that the design is governed by the condition, that vertical shear ½ *d* from face of cap is to be used for diagonal tension.

$$\text{Then} \quad \frac{S_1}{L} = \frac{4.50 + 2.07}{10} = .657$$

As this is practically the same value as that already used, the calculation will not be repeated, but the work should be done with the general formula (7).

If reinforcement is required for the entire vertical shear in excess of 40 lbs., use formula (9).

Then

$$Av_s = .027\ (.745 + .45)\,(10)\,(10) = 3.22 \text{ sq. ins.}$$

Bond Stress. For Square Deformed Bars at 100 lbs.

Opposite *B/L* used on Left Scale, find point directly above size of footing on lower scale *L*. Observe the next line of equal bond stress above this point. This represents the size of bar that will afford an average unit bond stress not greater than 100 lbs. In this case ¾″ square bars would be indicated.

Bent Bars. It has been shown that if bent bars are used, the area required amounts to .21 square inches, or 3% of the main tension bars. As these bars can be bent up with little increase of cost and as experiment shows much greater ultimate security with effective web reinforcement, it is always desirable to bend up an adequate number of bars to take the full estimated diagonal tension. The effect of this on the Bond Stress in the remaining bars must be determined.

Observe that the perimeter of ¾ bars is 3 (See Bar Data table), that of a ⅞ bar 3.5, and that the intersection of *B/L* and *L* is approximately one-fifth of the linear distance between the ¾″ and ⅞ lines, below the ¾ curve.

Then the perimeter of the bar that would be adapted to give precisely 100 lbs. adhesion is

$$3 + (1/5) \times (3.5 - 3), \text{ or } 3.1.$$

The next line above the ¾ line will probably provide sufficient surplus bond to allow of bending up the number desired without exceeding 100 lbs. in the remaining bars. The number of bars required for bond, of the size proposed for use, viz. ⅝", will be found accurately as that part of the number required for tension represented by a fraction having as numerator the perimeter of the bar to be used, and as denominator the perimeter of an imaginary bar adapted to the development of 100 lbs. bond at the B/L used.

Illustrating:

⅝ sq. bars required for tension, $7.15/.391 = 18.4$.

Bars required for bond, $(2.5/3.1)(18.4) = 14.4$.

Or $4/18.4 = 22\%$ of tension steel may be bent up. 19 bars will be used in tension, and 4 bars for diagonal tension.

The diagram of bending moments shows that for a considerable distance from face of cap, the bending moment is decreasing at the rate that makes it possible to dispense with this amount of steel 5" from face of cap. See page 7 for Bending Moment Diagram.

The calculation is very simple and is as follows:

$$2/3 \text{ of projection } C = 2/3 \text{ of } 33 = 22 \text{ inches.}$$

Then the rate at which the bending moment is decreasing per inch away from face of cap will be:

$$1/22 = 4\tfrac{1}{2}\% \text{ per inch, or } 22\tfrac{1}{2}\% \text{ in five inches.}$$

As one bar represents 1/19% of the area actually used, or 5¼%, the four bent bars will represent

$$(4)(5\tfrac{1}{4}) = 21\% \text{ bent up.}$$

Round Deformed Bars. If round bars are to be used, multiply the perimeter represented as the intersection of B/L and L by .7854. This gives an imaginary perimeter suited to the development of 100 lbs. of bond and to be used as the denominator of the fraction to be similarly determined as with square bars.

For example, $(3.1)(.7854) = 2.44$.

As 2.36 (See bar Data Table) is the perimeter of ¾ bars, ⅝ bars will probably provide the surplus bond required. Then

⅝ round bars required for tension, $7.13/.31 = 23$.

Bars required for bond, $(1.96/2.44)(23) = 18.5$.

Or $4.5/23 = 19\tfrac{1}{2}\%$ of tension steel may be bent up.

If this is not sufficient, use the next smaller size, and when a considerable amount of tension steel is to be bent up designate two points—for bending proportioned to the decrease in bending moment.

If small bars become objectionably numerous, resort may be had to vertical stirrups in place of bent bars, or to any desired combination of the two methods.

Above the curve for 3/8 square any type of mesh may be used rather than plain bars.

Plain Bars. As pointed out later, deformed bars at working stress of 100 lbs. give much better factors of safety than plain bars at 80 lbs., and the method of deriving the necessary results where plain bars are used is, therefore, given in connection with its use for analysis in Part 7.

Bond Stress. Bond stress in bent bars may be provided for by extending the bar to grip length beyond the 45° line of diagonal tension fracture equivalent to its tension value, but the cheaper, easier and safer plan is to specify that all bent bars be hooked at the ends with standard hooks, which are capable of developing the tension value of the rods.

Stirrups. Talbot concludes that .6 the depth from the surface of the concrete may be used as part of length for grip adhesion. This length plus any added bends or hooks at the top must supply resistance to bond stress equivalent to tensile value of the stirrups.

Areas, perimeters and tension values are given in table of bar data.

If stirrups are made continuous, bond stress becomes negligible.

Design. The use of the diagrams for the design of a typical square footing will now be illustrated without repeating same explanations.

Soil load 5000 lbs. per sq. ft., column load and footing weight 390,000, carried by spiralled concrete column 22″ square.

$$\text{Then } L = \sqrt{\frac{390{,}000}{5000}} = 8.83 \text{ ft.}$$

Make cap 6″ wider than column.

Then

Ratios $E/L = 22''/8.83 \times 12 = .208.$

$B/L = 34/106 = .32.$

$H = C\,K\,L = (2.72)\,(.14)\,(8.83) = 3.36.$

$d = C\,K\,L = (1.66)\,(.14)\,(8.83) = 2.06.$

$D = 2.06 + .25 = 2.31.$

$I = H - d = 3.36 - 2.06 = 1.30.$

The minimum depth for *I*, or depth of cap, will always be determined by the limit shear, as 120, at the face of column, through the combined depth of cap and footing.

The maximum depth for *I* will be any thickness in excess of this minimum required by code provisions or Building Department interpretation.

$$M = 2.83 + 4.12 + \frac{8.83 - 6.95}{2} = 7.89 \text{ ft.}$$

$$A_s = PA = (.90)(5.60) = 5.04 \text{ sq. ins.}$$

Intersection of $B/L = .32$ and $L = 8.83$ is 4/10 below the ¾″ curve. Perimeter value for 100 lbs. bond is, therefore, 3.4. ⅝ bars with 2.5 perimeter and area of .39 will be used. Then

Bars required for tension, $5.04/.39 = 12.9$.

Bars required for bond, $(2.5/3.4)(12.9) = 9.5$.

Then 3.4 bars, or 27% of steel, may be bent up.

13 ⅝ square deformed bars 8′ 6″ long will be used each way, 3 of which, or 23.6%, will be bent up.

Width of projection

$$C = \frac{8.83 - 2.83}{2} = 3'\ 0'' = 36''$$

2/3 of $C = 24''$. Then from Bending Moment Diagram, bending moment diminishes $1/24 = 4.17\%$ per inch.

Then $23.6/4.17 = 5\ 2/3$, or say 6″, the distance from face of cap, where 3 bars may be bent up.

Ordinance allows point at distance jd from face of cap to be used as place to measure vertical shear for diagonal tension, or

$$(7/8)(2.06) = 1.81 \text{ ft. from face of cap.}$$

Opposite B/L .32 on Curve S find point above .635 on Lower Scale.

Then point of 40 lbs. shear occurs at

$$\frac{(.635 \times 8.83) - 2.83}{2} = 1.39'$$

from face of cap, and shear reinforcement will not be required by ordinance, but will be used as before figured, as it practically costs nothing, and adds much to security.

Bond in Bent Bars. Using Formula (13)

Available grip length

$1.42(2.04 - 2'') - .7(2.04 - 6'')$

$1.42(1.88) - .7(1.54) = 2.67 - 1.08 = 1.59.$

$1.59 = 19''$. Then $\dfrac{19}{5/8} = 30$ bar diameters.

As 40 bar diameters are required for bond, an additional length of 10″ bent down parallel with upper surface of footing may be used, or Standard hooks requiring a bend of 180° through a diameter of 3″ may be specified. The hooks and a short tangent at ends will require 6″ in length of rods.

The length of the three bars to be bent up may now be estimated as follows:

Width of cap	2.83
$2X=(2)(6'')=$ Dist. to bend from cap . .	1.00
2×2.67 inclined part as above	5.34
2 hooks @ 6″ each	1.00
Total length	10.17

Tension bars for width

$$O = \frac{8.83 - 7.87}{2} = .48$$

$(\frac{3}{4})(.48/8.83)(5.04)=.21$ sq. ins.

Use ½″ square deformed bars area = 0.25.

Concrete Stress.

As intersection of B/L .32 and soil pressure 5000 occurs below line Z, this will be less than 650 lbs. and need not be figured.

No stirrups required.

Spacing of tension bars:

One bar at either end of M leaves 12 intermediate spaces, or $7.87/12=.656$, or slightly less than 8″.

Bars in width O will be centered or placed half way between last bar in M and edge of footing, leaving about 3″ from these ½ bars to face of concrete.

For further information as to the application of diagrams to conditions not explained, consult the last section of this booklet.

USE OF DIAGRAMS FOR REVIEW.

Building Departments.

The adaptation of these diagrams to review assumes the ready determination of stresses existing in designs presented for approval.

The examining authority is only interested in assuring itself that the code limits are respected, and is not at all concerned as to how wasteful the design may be within those limits.

The methods presented, therefore, are directed to the determination of correct dimensions for the loads assumed, and comparison of these with actual dimensions of the design.

Examples of actual footings selected at random from designs taken from Building Department records of Western cities will serve to illustrate the facility with which the desired information may be secured by the aid of diagrams.

An official report blank covering the points to be investigated will insure systematic record of results, and avoid the omission of anything requiring consideration.

Such a blank adapted to local needs with regard to working stresses used will contain place for sketch and heading as shown, and for any other desired data.

No. 1. Foundation Report............................Building.

Date.................... Street.....................

Owner .... Architect.................

SKETCHES.

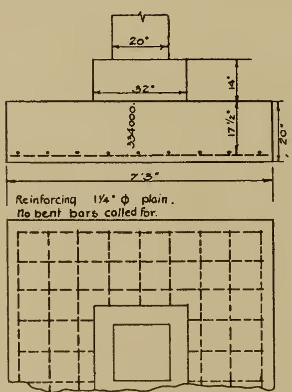


FIG. 3. REVIEW OF COL. E 5

.... BUILDING.

Footing No. E 5.

O. K.

Soil pressure 6350 lbs. per sq. ft. Value of $K = .18$.

Size.

$$L = \sqrt{\frac{334000}{6350}} = 7' \ 3'' \text{ O. K.}$$

Depths. $E/L = (20/7'\,3'' \times 12) = .23.\ B/L = (2.67/7.25) = .368.$

Cap and Footing. $H = C\,K\,L = (2.44)\,(.18)\,(7.25) = 3.18$

Design 2.63 Inc.

$d = C\,K\,L = (1.39)\,(.18)\,(7.25) = 1.83$

Design 1.46 Inc.

Shear at Face of Column. (3.18/2.63) (120) = 145 Decrease.

Shear at Face of Cap. (1.83/1.46) (120) = 149 Decrease.

Area of Steel. $A_s = PA = (.955)\,(3.8) = 3.63$ sq. ins.

Design, 9 plain Ø 1¼″ each way, (9) (1.23) = 11.04 O.K.

Width M.

$$(2.67) + (2)\ (1.83) + \frac{7.25 - 6.31}{2} = 6.79 \text{ ft.}$$

Bond. From Diagram, intersection of $B/L = .368$ and $L = 7.25$ occurs on bond line for ⅝ square bars with 2½″ perimeter for 100 lbs.

Correct perimeter for round bars at 80 lbs.

(2.5) (80/100) (.7854) = 1.57, or ½″ round bars.

See Bar Data Table.

Perimeter of bar used in design, 3.94 for 1¼″ Ø. Area 1.23.

Then bars required for tension 3.63/1.23 = 2.95 bars.

Bars required for bond, (3.94/1.57) (2.93) = 7.40 bars.

As 7.4 bars are required in width M, and at least 1 bar must be placed on each side in width O, it appears that 9.4 bars are needed, and that no surplus bond permitting bending of part of bars exists.

As no significant expense is caused by requiring bent bars, the design should be rejected.

This was at once apparent from the total area of steel and size of bars used. The calculations have only been continued to show the use of diagrams in analysis of an uneconomical design.

The data will now be used to illustrate diagram bond design, and also economy arising through the selection of type of bar.

Since the correct perimeter for plain Ø bars at 80 lbs. is shown to be 1.57, and bent bars are required, the next size will be used, viz., ⅜ Ø having an area of .11 sq. ins. and a perimeter of 1.18 ins.

Then bars required for tension, 3.63/.11 = 33.

Bars required for bond, (1.18/1.57) (33) = 24.8.

8.2/33%, or 25% of steel in width *M*, may be bent up.

Then using 33 bars for 32 spaces, the centering will be 2.55 inches.

While the allowable interval between bars (as fixed by the Joint Committee) is not exceeded in this case and will rarely be reached in footings, the large number of bars increase expense of placement, and involve higher tonnage price.

This leads to the general conclusion that square deformed bars represent maximum economy, under present conditions of allowable working stresses, and the actual relations of areas and perimeters. See page 57 for verification.

The results of substituting square bars mark the contrast.

Correct perimeter 2.5.

Use next size perimeter 2. Area .25.

Bars required for tension, 3.63/.25 = 14.5.

Bars required for bond, (2/2.5) (14.5) = 11.6.

This requires the handling of 15 bars each way in place of 32, with a direct saving of about 50% of placement labor, a slight saving in tonnage cost due to using a ½″ square instead of a ⅜ Ø, and a possible small increase in using deformed instead of plain sections. In the San Francisco market no differential is now made between plain, twisted and deformed bars.

Also a more satisfactory centering is secured, viz., 15 bars for width *M* with 14 spaces @ 5.8″ centers.

Diagonal Tension. If diagonal tension were now investigated on the basis of the fact that the city ordinances permit vertical shear at distance of ½*d* from face of cap to be used as index of diagonal tension, the following results would be obtained:

From Diagram: Opposite $B/L = .368$, find point on curve S vertically above answer .68 on Lower Scale S, being value of $\frac{S}{L}$

Value of

$$S_1 = B + \frac{2d}{2} = 2.67 + 1.82 = 4.49$$

$$\text{Then } \frac{S_1}{L} = \frac{4.49}{7.25} = .62$$

For External Shear. Opposite .62 on Left Scale, find point on curve V vertically above .153, the answer on Lower Scale V.

Then $V = V^{\Delta} W = (.153)(334000) = 51200$ lbs.

$$v = \frac{51200}{(4.49)(12)(7/8)(1.83)} = 60 \text{ lbs.}$$

As 40 lbs. is allowable in concrete, design will call for reinforcement for 20 lbs. in steel and Formula (8) may be used.

$$A_{vs} = \frac{(.68 + .62)(7.25^2)}{444} = .16 \text{ sq.ins., cr}$$

$.16/3.63 = 4.4\%$ of tension reinforcement.

Previous calculations show that 2.9/14.5, or 20%, of steel may be bent up, or practically four times the amount required, for vertical steel.

Bending moment diagram shows that 20% of steel may be bent up 4 inches from face of cap.

Calculation for this information follows:

$$C = \frac{7.25 - 2.67}{2} = 2.29 = 27\tfrac{1}{2}''.$$

$2/3$ of $C = 18.33$ inches.

Inspection of bending moment diagram shows that bending moment lessens at such a rate that it will have decreased $1/18.33 = 5.4\%$ per inch away from face of cap. The distance away from projection for 20% of steel to be bent up will then be $20/5.4 = 3.7$ inches, say 4 inches.

Talbot's experiments show that this steel should be used to guard against failure by diagonal tension, which may occur in reinforced footings, when the external loads are such as to produce elastic limit stresses in the tension steel.

At working stresses there is no stress whatever in the diagonal tension steel, for the concrete carries these stresses until incipient cracks begin to transfer them to the steel.

It follows, therefore, that the most advantageous location for such steel is normal to the plane of fracture and midway in its length, or on the plane of symmetry passing through the center of the footing section.

This would imply bending at the vertical line passing through the face of the cap, but as this is impossible, and as the anchorage does not extend to the top of the footing, the point of symmetry is really below the center several inches.

It will be seen from Figure (2) that the condition as to anchorage is improved as the point of bending is removed from the edge of cap, and as long as the steel crosses the line of fracture fairly near the center of the effective depth, its placement may be considered satisfactory.

In actual practice, the investigation need not have continued beyond the discovery that the depth of footing is deficient, for this relation is fundamental since it enters into all subsequent calculations.

It follows, therefore, that footings may be of any depth involving

less than 120 lbs. of shear, but that no footing can be thinner than is requisite for the allowable limit stress of 120 lbs.

The same data will now be used in modified form to indicate the effect on review work, in case the vertical shear is less than 120 lbs.

No. 2. For same footing, assume that design shows:

$H = 3.25.$

$d = 2.25.$

$A_s = 7$ ½″ sq. deformed straight bars, width M.

3 ½″ sq. bent bars, width N, bent up at 45° 6″ from cap, without hooks.

1 ½″ sq. bar, width O.

Examination.

Soil pressure 6350 O. K. for locality.

Size of footing 7′ 3″ O. K. for load.

$H = C\,K\,L = (2.44)\,(.18)\,(7.25) = 3.18$

Design 3.25 O. K.

$d = C\,K\,L = (1.39)\,(.18)\,(7.25) = 1.82$

Design 2.25 O. K.

Shear less than 120. O. K.

Area of steel (in proportion to depths)—

$A_s = (1.82/2.25)\,(.955)\,(3.8) = 2.94$ sq. ins.

Available 10 bars @ .25 2.50 Increase.

The steel used is about 15% less than the Talbot assumptions, and evidently involves bending moment assumptions greater than those shown in the comparative diagrams given in Part 5.

If other than the Talbot assumptions are allowed, the diagram results should be affected by a multiplier representing the divergence of the assumptions. This may be used as a constant average percentage for all moments, if figured for $B/L = .42$.

In this case the diagram results are used, and 2 more bars will be required.

Bond. Diagram intersection ⅝ bars, 2.5 perimeter.

Use ½″ bars, 2 ins.

Bars for tension, $2.94/.25 = 11.8$.

Bars for bond, $(2/2.5)\,(11.8) = 9.45$.

Used in design, 10.

Straight, 7. Increase 2.

Bent, 3. O. K.

Distribution. Respace 12 bars in width $M = 6.78$ ft. O. K.

Steel for Width O. $\frac{7.25 - 6.78}{2} = .23$ ft.

Nothing required.

Available 1 ½″ bar. O. K.

Diagonal Tension. ½*d* from cap.

$S_1 = 2.67 + 2.25 = 4.92.$

$S = (S/L)(L) = (.68)(7.25) = 4.92.$

As the point of 40 lb. shear coincides with the ordinance position for measuring diagonal tension, bent bars would not be required, but the Department has ruled that the point of 60 lb. of vertical shear shall be regarded as the allowable minimum for figuring diagonal tension with web reinforcement provided for 20 lbs. of this amount.

Then using Formula (8):

From diagram for $B/L = .368.$

$S/L = .68.$ $S_1/L = .58.$

$$A_{v_s} = \frac{(.58 + .68)(7.25)^2}{444} = .15 \text{ sq. ins.}$$

Amount to be bent up in revised design: 3 ½″ bars @ .25 = .75 sq. ins., or 5 times amount required. O. K.

Point of Bending. Percentage of steel to be bent up, 3/12 = 25%.

Bending moment diagram shows that this may be bent up 5 inches from cap.

2/3 C = 18.33 ins. Then 1/18.33 = 5.45 %.

25/5.45 = 4.6″, say 5″, from face of cap.
Design, 6″. Reduce to 5″.

Bars shown terminating at point 2″ below top of footing. Clearance O. K.

Acting Length of Embedment.

(1.42)(2.25 — .16) — 0.7(2.25 — .29) = 1.60 = 19¼″.
19.25/0.5 bar diameter = 38½.

Required (Table of Bar Data), 40.

Bars must be extended horizontally or provided with standard hooks.

It will be seen that most of work involved thus far has been due to explanation rather than review.

Another footing from the same city will now be used:

SKETCH.

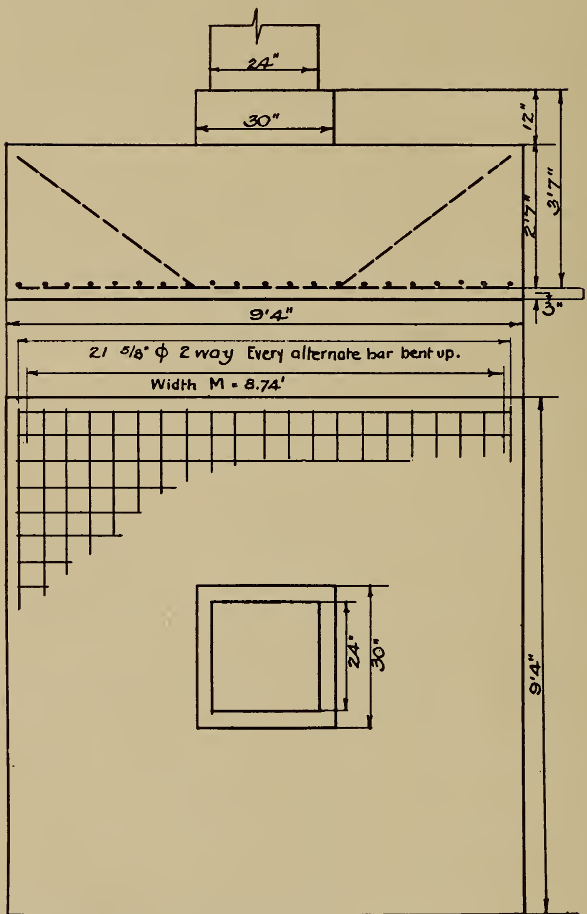


FIG. 4. REVIEW OF COL. 3 ....BUILDING.

No. 3. $L = \sqrt{\frac{461{,}000}{5300}} = 9'\,4''$. O. K.

$E/L = 24/112 = .214$ $\quad B/L = 30/112 = .268$

$H = (2.65)\,(.15)\,(9.33) = 3.70$

Design 3.58 Increase.

$d = (2.06)\,(.15)\,(9.33) = 2.85$

Design 2.63 Increase.

Shear at cap = (2.85/2.63) (120) = 130 lbs. Decrease.

$A_s = (.82)\,(6.25) =$ 5.12

Design 21 ⅝ Ø @ .31 = 6.52 O. K.

Spaced 5½" ¢ each way.

Every alternate bar bent up.

$M = 2.50 + 5.66 + .58 = 8.74$.

Available in width M, $8.74/.5 = 19$.

As 10 bars are bent up at point of maximum bending moment, only 9 bars are available for tension $= 2.79$ sq. ins. Required, 5.12 sq. ins. Increase.

Deficiency in tension steel can be corrected by proper placement of available material.

To ascertain correct distribution, investigate bond conditions.

From Diagram, bond intersection for $B/L = .268$ and $L = 9'\ 4''$ occurs at—

$7/8 + 2/3 = 3.50 + .33 = 3.83$ perimeter for square deformed bars @ 100 lbs.

For round deformed bars @ 100 lbs. the correct perimeter would be $(3.83)(.7854) = 3.01$.

Design, 5/8 Ø deformed bars with perimeter of 1.96.

Bars required for tension, $5.12/.31 = 16.5$.

Bars for bond, $(1.96/3.01)(16.5) = 10.7$.

Reduce number of bent bars and change point of bending.

As there are 19 bars available for tension in M, and only 11 required for bond, 8 alternate bars near middle of footing may be bent up 9 inches from cap. $C = \frac{9'4 - 2'6}{2} = 3'5 = 41''$

$2/3\ C = 27\ 1/3$ inches.

$1/27.33 = 3.66\%$ per inch decrease of tension.

Steel decreases $5.5/16.5 = 33.4\%$.

Then $33.4/3.66 = 9$ inches.

That is, there must be 11 bars available for bond at a point 9″ from face of cap.

Diagonal tension for ½d from cap, as per ordinance.

$S_1 = 2.50 + 2.83 = 5.33$. O. K.

$$S = \left(\frac{S}{L}\right)(L) = (.58)\ (9.33) = \quad 5.41$$

As S_1 practically coincides with point of 40 lbs. shear, only minimum reinforcement for 20 lbs. in steel in excess of 40 lbs. allowed in concrete will be required.

This need not be figured, as the bent bars will provide several times more resistance to diagonal tension than will be required.

Grip Length—Bent Bars. Formula (13).

$1.42(d - c) - 0.7(d - X)$.

$1.42(2.83 - .16) - 0.7(2.83 - .42)$.

$3.79 - 1.69 = 2.10 = 25\frac{1}{4}''$.

Diameters of bars—

$25.25/.625 = 40\frac{1}{2}$ diameters. O. K.

Critical Points. Examination should be made as to the following critical points:

Limit shear at edge of concrete columns, C. I. base, or steel slab.
Limit shear at edge of cap.
Bond in tension bars beyond point of bending.
Bond in bent bars above line of rupture in diagonal tension.
Bending moment at point where bars are bent up.

Experiment.

For the purpose of illustrating the use of the diagrams for the calculation of ultimate stresses from experimental data, some of the footings of the Talbot tests will be used.

Since the diagrams are based on constant values of $j=.875$, $fs=16000$, $v=120$ max., and $u=100$ for square deformed bars, it is evident that other values of these factors may be obtained by proper use of diagram results.

The first two footings encountered on page 74 of Bulletin 67 will be used, as follows:

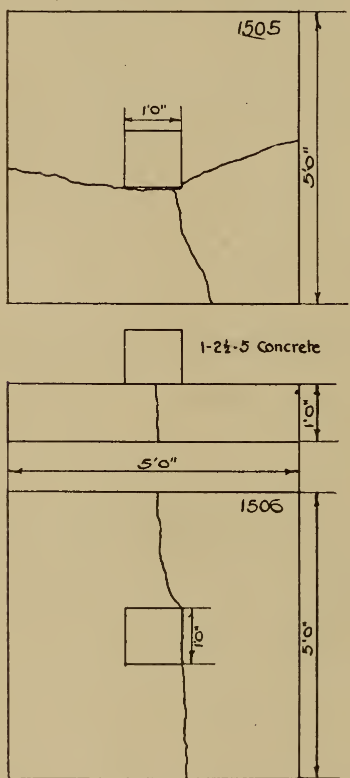


Fig. 6. Unreinforced Concrete Footings—Cracks at Failure. *(Talbot.)*

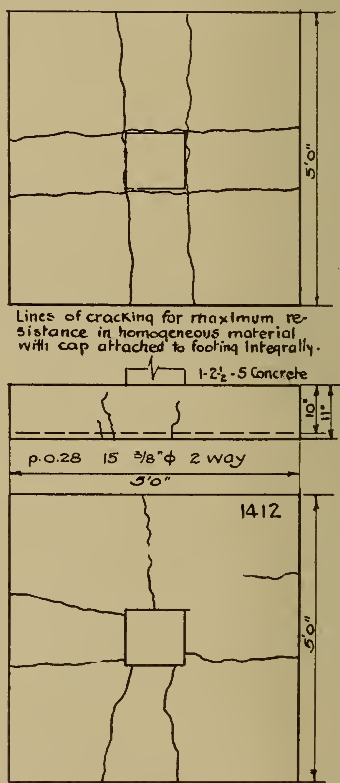


Fig. 5. Illustrating Cracks on Bottom of Reinforced Concrete Footing — Typical Tension Failure. *(Talbot.)*

Footing No.	Description.	Per Cent.	Load at Failure.	Depth Over All.	Depth to Center of Steel.	Cause of Failure.	Size of Foot.	Size of Cap.
1411	15 3/8 Ø plain	0.28	112,000	11	10	Tension	5′ sq.	1′ sq.
1412	"	0.28	160,000	11	10	Tension	5′ sq.	1′ sq.

Value of j used in Talbot calculations, for p, 0.28 = .90.

Calculated Values.

For No. 1411. Soil value 112,000/25 = 4500 lbs. $B/L = 1/5 = .2$. B/L at $1d = 2.65/5 = .53$.

For No. 1412. Soil value 160,000/25 = 6400 lbs. $B/L = 1/5 = .2$. B/L at $1d = .53$.

For 1411. If cap width were 2.67 there would be vertical shear of 120 lbs. at its face for a depth

$$C\,K\,L = (.795)\,(.127)\,(5) = .5.$$

Actual depth 10″ = 0.83.

Then actual shear $1d$ from face of cap—

$$(.5/.83)\,(120)\,(.875/.90) = 70 \text{ lbs.}$$

Talbot determination 69 lbs.

For 1412. $d = C\,K\,L = (.795)\,(.18)\,(5) = .71.$

$$v = (.71/.83)\,(120)\,(.875/.90) = 100 \text{ lbs.}$$

Talbot determination 99 lbs.

It will be seen from the figures that the determination of the depth of a footing having a cap of same size as point selected to measure vertical shear makes it possible to determine the actual shear as the same ratio of 120 lbs. that exists between the depths. Also, since the constant j entered into the original calculation as a divisor, it must now be put back as a multiplier, and the result divided by the same value of j used in the calculations for the experiments.

It will be interesting to calculate the shear at the cap instead of $1d$ away.

For 1411. $C\,K\,L$ for cap ratio $.2 = (2.84)\,(.127)\,(5) = 1.8$

1412. $C\,K\,L$ for cap ratio $.2 = (2.84)\,(.18)\,(5) = 2.55$

Then for 1411. $v = (1.8/.83)\,(120)\,(.875/.90) = 252$ lbs.

1412. $v = (2.55/.83)\,(120)\,(.875/.90) = 356$ lbs.

Tension in Steel. Steel required.

Above 5 on Lower Scale L, find point on curve A opposite 1.8, the answer on Right Scale.

Opposite B/L .2 on Left Scale, find point on curve P above .675, the answer on Lower Scale P.

Then $A_s = P\,A = (1.8)\,(.675) = 1.21$ sq. ins.

Used 15 3/8 Ø @ .11 = 1.65.

Considered as effective by Talbot: (46/60) (1.65) = 1.265 sq. ins.

Then 1411. $f_s = (1.8\ /0.83)\,(16000)\,(1.21/1.265)\,(.875/90) = 32200$
Talbot 31900

1412. $f_s = (2.55/0.83)\,(16000)\,(1.21/1.265)\,(.875/90) = 45800$
Talbot 45600

Bond Stress. External shear $V = V^{\Delta}\ W$.

Then for 1411, opposite $B/L = .20$ find point on curve V above .24, the answer on Lower Scale V.

$$\text{Then } v = \frac{.24 \times 112{,}000}{(15)(.765)(1.18)(10)(.9)} = 220$$

Talbot determination 220

Similarly for 1412, since all the factors are identical, except the loads, the stresses will be as the loads, or

(160,000/112,000) (220) = 314

Talbot determination 314

Illustrating the value of average bond stress in contrast with maximum stress used by Talbot:

From diagram, intersection of $B/L = .20$ and $L = 5$ occurs 2/5ths below the ½″ line, or (2/5) × (.5) = .2 + 2 = 2.2 perimeter value for square deformed bars at 100 lbs.

For plain rounds at 80 lbs. value:

(2.2) (80/100) (.7854) = 1.38

Bars used, ⅜ Ø 1.18.

Then for 1411—

(1.18/1.38) (80) (1.8/0.83) (.875/.90) (1.21/1.265) = 136 lbs.

Similarly for 1412, since all the factors are identical, except the depths, the stress will be as the depths, or

(2.55/1.8) (136) = 194 lbs.

To check these calculations for the average bond, assume that tension steel had clearance at ends of 2″, or bars were 60 — 4 = 56″ long. Then

(56″ — 12)/2 = 22″ = effective bond length.

Tension capacity of one ⅜ Ø @ 31850 lbs,

(1.18) (1.18) (1273) (31850/16000) = 3530 lbs.

Bond 3530/(1.18 × 22) = 136 lbs.

Tension capacity of one ⅜ Ø @ 45400 lbs.

(1.18) (1.18) (1273) (45400/16000) = 5040 lbs.

Bond 5040/(1.18 × 22) = 194 lbs.

The ratios between maximum and average bond are shown by figuring that the bars extend to the face of the footing, since the external shear is used to that point in calculating the maximum stress.

Then $5040/(1.18 \times 24) = 177.5$.

$$\text{Then} \quad \frac{\text{Maximum Stress}}{\text{Average Stress}} = \frac{314.}{177.5} = 1.765$$

This ratio varies from 1.5 to 1.77 in the large number of instances examined in connection with the diagrams.

It will, of course, vary with the length of clearance at the end of the bars and should apparently be 1.5, when the bars extend to the face of footing on account of the parabolic form of the curve defining the distribution of bond.

It will be seen that critical maximum bond stresses are from 1.5 to 1.77 times as great as the average working stresses used in design.

When the dimensional constants are worked out for given experimental information, and the user of the diagrams has become somewhat familiar with the methods of their application, it will be seen that rapid results can be secured.

The calculations of these and many other experiments have been used to check the accuracy of the diagrams, and the results have invariably proven within a small margin of error.

General Information.

The data contained in the following pages is intended to make available to beginners in the art the contrasts found in practice between fundamental assumptions, the relation between working stresses and ultimate rupture loads, as shown by experiment, and to interpret to some extent the experimental results used.

It is substantially the matter prepared for a lantern slide talk before the Architectural Club of San Francisco, an association of architects and designers, and covers many points of detail in a more elementary manner than would be necessary if intended only for an audience of engineers.

Nevertheless, it is believed that many designers will find this form of presentation useful, and the information is given in such a manner as to enable anyone to make his own interpretation.

The explanations are, therefore, addressed only to those who find them useful.

Where specific methods are recommended, as in the case of bending moment and shear calculations, it should be understood that this is only the author's own preference, and that formulae are invariably given in such general form as to permit the use of diagrams with any assumption desired.

Caps. The questions relating to the dimensions of caps are usually assumed to be so simple as to require no more thought on the part of the designer than the use of a 60-degree triangle to establish the depth for an assumed projection beyond the edge of column or other loading element.

One of the concrete specialty companies has been largely followed in making the projection 6″ for columns under 24″ in diameter, and 8″ for columns larger than 24″, with depths of twice the projection.

If the cap is poured separately from the footing, it will be possible to determine the stresses in bending and shear. For this purpose a 24″ hooped column will be used with a load of 450,000 lbs. and a cap 36x36x12″.

The modulus of rupture for the concrete in the cap will be determined by the formula:

$$M = sS$$

The value of $E/L = 24/36 = .666$

Then from diagram for $E/L = .666$,

$$M = (.12)(450{,}000)(3)(12)/10 = 194{,}200$$

$$S = (36)(12)(12)/6 = 864$$

Then $s = 194{,}200/864 = 225$ lbs. per sq. inch.

Talbot tested a series of concrete footings without reinforcement tabulated on page 73 of Bulletin 67.

Two footings of same depth as the cap just figured are selected for illustration. See Fig. 6.

No.	Age Days.	Load at Failure.	Mod. of Rupture.	Control Beams Mod. of Rupture.	Control Beams Age.	6" Cubes Max. Load per Sq. In.	6" Cubes Age.
1505	86	86,000	195	376	95	3618	128
1506	77	67,000	152	272	99	2260	116

Checking the modulus calculated $B/L = 1/5 = .2$

For 1505. M^{Δ} from diagram for $B/L = .20 = .544$

Then $M = (.544)(86{,}000)(5)(12)/10 = 281{,}000$

$S = (60)(12)(12)/6 = 1440$

$s = 281{,}000/1440 = 195$ lbs.

For 1506. s will evidently be as the loads, or

$s = (67{,}000/86{,}000)(195) = 152$ lbs.

Since the conditions of mix and other circumstances were identical, there must be a rational explanation as to why No. 1506 failed at a smaller load than 1505 and why both failed at smaller unit stresses than was the case with control beams.

It should be remembered, in the case of the footings tested, that there is a sudden change in the intensity of the tension stress at the face of the cap, because it is integrally a part of the footing.

Referring to Footing 1506, which failed in a single line of weakness, it would seem evident that the failure might have occurred in either of four directions as shown in Figure 5. Then if the material were absolutely homogeneous it would resist rupture until failure occurred simultaneously on all four lines, assuming that the combined resistance in these directions were still the critical weakness of the footing.

If this were roughly so, the resistance developed in 1505 should be 1.4 times as great as that developed in 1506, or its modulus of rupture would be:

$$(1.4)(152) = 211 \text{ lbs.}$$

The average value assigned the modulus of rupture for concrete as determined by control tests is 400 lbs. and effort to rationalize the behavior of a material showing such a disproportionate critical weakness as is exhibited in the low modulus of 152 lbs., at which 1506 failed, is futile.

This merely emphasizes the fact that, where concrete is subjected to bending stress without reinforcement, safety factors should be taken as a proportion of the critical weakness rather than as a proportion of the average value of the modulus secured by small tests.

Returning to the cap considered, two conclusions seem possible:

First: That in a rough way the attachment of the cap to the footing doubles the resistance of the footing, if of homogeneous material, since it forces ultimate failure in a perfect material on four lines, instead of on two lines in which failure would occur if the cap is free from the footing.

Second: Caps are subjected to very little bending in actual practice, as it is evident that if the footing yielded in the same degree as the bed of springs in which footings were tested, the concrete in the cap could not resist the bending stresses resulting.

If these conclusions are accepted, the importance of monolithic construction between the cap and footing will be appreciated.

The vertical shear at face of column, considering the caps as separated from the footing, will amount to 246 lbs. per square inch, as follows:

From diagram for $B/L = 2/3 = .67$

$$C\,K\,L = (.48)\,(.00002834)\,(50{,}000)\,(3) = 2.05$$

$$\text{Then } (2.05/1.0)\,(120) = 246 \text{ lbs.}$$

As the value of K in the above example is outside the diagram limits, the soil value, $450{,}000/9 = 50{,}000$, has been multiplied by its increment value and so extended to the point used.

This again illustrates the fact that the cap should be constructed monolithic with the footing, and figured for shear at face of column by considering the combined depth of cap and footing as affording resistance equivalent to external shear within working stress limits. This constitutes the basis of design in the diagrams developed.

It assumes that the compression is distributed through the cap with an increasing reduction of unit stress as depth increases, but that the result is merely comparable to increasing the size of a column under the same loading. Somewhere below the point of such change of section, unit stress might again be considered as uniform across the entire section, but at intermediate points the outer fibers of the column would carry very little stress.

The cap might also be considered in a way as analogous to a butt plate in steel construction.

In practice the limiting depth of the cap is apt to be some proportion of the projection allowed by local ordinances or interpretation.

The rulings on this point in San Francisco agree with the method herein used, involving the assumption that the depth of the cap is independent of the width of projection and is determined by limit shear only.

This provides minimum depth, which will have to be increased, if so required by Building Department interpretations elsewhere.

In general, the condition of maximum economy will be realized when cap width is such that $B/L = .60$ and cap depth is 4/3 of cap projection.

Depth of Footings. Custom has permitted punching shear around the perimeters of footing caps of from 100 to 160 lbs.

The latest report of the Joint Committee, recognized as the highest authority on the subject, allows 120 lbs. for 2000 lbs. concrete "when all tension normal to the shearing plane is provided for by reinforcement."

If this unit stress is respected, it follows that, with the exception of very low soil values and small footings, the depth of all footings will be governed by the allowed unit stress in shear.

As ultimate economy is always in the vicinity of limit stresses, it becomes logical to use 120 lbs. as the governing factor in design.

The failure of footings by direct shear is impossible, as Talbot's experiments show failure by diagonal tension at much smaller loads than would be required for direct shear. The factor of safety for pure shear is probably not less than 10, which assumes a rupture value in shear of 1200 lbs. per square inch.

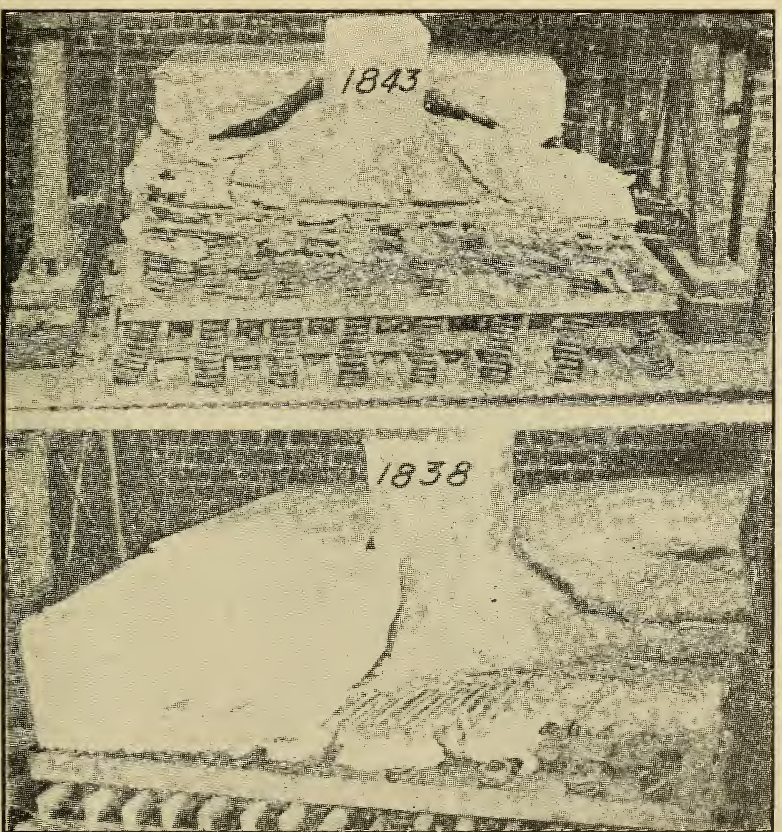


FIG. 7. ILLUSTRATING DIAGONAL TENSION FAILURE. *(Talbot.)*

Nevertheless working limits, when made by competent authority, should be respected, and approval of plans should be based on keeping stresses within limits assigned by such authority.

It follows that intelligent designers should use the full value of limits fixed. Employers should not be satisfied to find, in reviewing design work, that excess strength has been provided, as is often the case where the checker O. K.'s the results, but should insist that proportions be so related that limit stresses in all material will be availed of, as closely as possible.

It will be seen at once that the adoption of a constant maximum shear of 120 lbs. means that the ratio of depth to length, or d/L, will change with every change of cap size.

Good designers have used what was considered an average value of $d/L = .20$ as a constant, but consideration of the data in the diagrams will show that the range of this factor, far from being constant, varies with each change of B/L.

For instance, for a cap ratio of .48 and a soil pressure of 12,000 lbs. the values of C and K are .94 and .343 respectively. Then d/L for these conditions equals $(.94)(.343) = .322$.

The same data for low soil values and small cap ratios give the following results:

For cap ratio of .35 and soil pressure of 2000 lbs. the figures are:

$$d/L = (1.46) \times (.057) = .083$$

It should, therefore, be remembered that the basis of design in these diagrams is the maintenance of 120 lbs. of vertical shear through the footings at the face of all caps.

On the diagram will be found a line marked Z, above which the limiting factor in the determination of depth will be the compressive stress in the concrete.

The intersection of a vertical line from the Soil Value Scale with line Z indicates the highest ratio of cap width B/L which can be used with that soil value without exceeding 650 lbs. in the concrete, or, *vice versa,* the intersection of any given B/L with line Z is immediately under the limit soil value for that B/L.

It is accurately established for footings of medium width of about 10 feet, and is substantially correct for all sizes. It will be found that depth of very small footings will be determined by some arbitrary minimum depth fixed by practical conditions, such as the depth of a form-board, or 6″.

This line makes it possible, by mere inspection of known factors, to determine at once whether concrete stress should be investigated.

To illustrate the saving of work accomplished by this, a case will be assumed where the intersection comes above the line, and depth is, therefore, to be determined by concrete stress:

Column load 150,000. Soil value 1500. $B/L = .46$.

$$L = \sqrt{\frac{150{,}000}{1500}} = 10'$$

Shear depth, $C\,K\,L = (1.00)\,(.0425)\,(10) = .425$.
As d_c must be greater than this, .6 will be assumed temporarily.
Then $b = 4.60 + 1.20 + (10 - 5.80)/2 = 7.9$.
Bending moment from diagram,

$$M = (.287)\,(150{,}000 \times 10/10) = 43{,}000$$

Then by Formula (14)

$$d^2 = \frac{2 \times 43{,}000}{(3/8 \times 7/8)\,(7.9)\,(144)\,(650)} = .353$$

$$d = \sqrt{.353} = .595$$

It will be seen that the work involved is much more troublesome than the method using shear as the governing factor.

One of the troublesome elements is the fact that d is a factor in the determination of width b, which in turn is used to determine d. This makes it necessary to cut and try.

Bending Moments. For comparison different formulae in general use by competent, responsible designers have been applied to a footing having a common ratio of $B/L = .42$, selected at random.

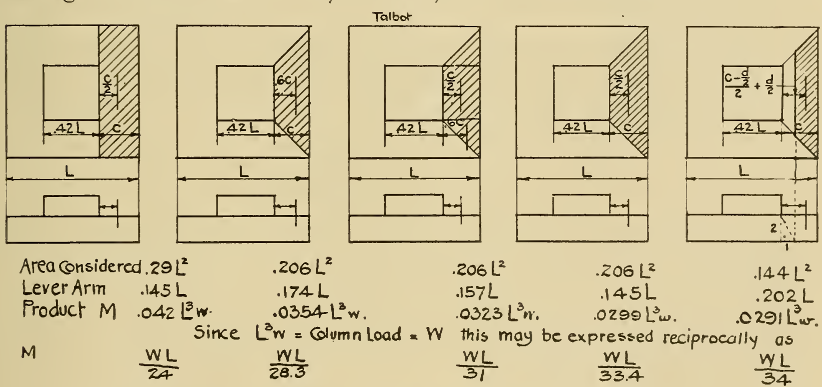


FIG. 8. ILLUSTRATING BENDING MOMENT ASSUMPTIONS.

It will be seen that, while the assumptions as to area considered vary as much as one hundred per cent., the resulting distance between the critical point at the face of the cap and the center of action assumed for the load tends to lessen this discrepancy. Nevertheless, there is a difference between extremes in the final result equal to fifty per cent.

It is also proper to say that more footings are probably in use. designed on the basis of the smallest value, representing the practice of one of the largest manufacturers of reinforcing products, than is true of any other type illustrated. The assumption in this formula, that the critical point for maximum bending stresses in the steel is on the line of the base of a frustrum of a pyramid, where rupture in diagonal tension would occur, appears entirely reasonable. The same reasoning is used by Talbot as the basis of selecting a point for the measurement of diagonal tension.

Inasmuch, however, as there is an experimental basis for the Talbot formula, lacking in the others, it has been used in these diagrams throughout. Anyone desiring to use other values can do so by affecting the value of A_s found from diagram with a coefficient representing the divergence of his formula from the one used in terms of a constant percentage, more or less than A_s.

Resisting Moments. The resisting moment in the steel has been used in the diagrams, or

$$M = A_s f_s jd$$

Equating this with the bending moment and solving for steel area, we have:

$$A_s = \frac{M}{f_s\ jd}$$

Where steel is uniformly distributed both ways through a square footing, the stress in the outside bars is less than in those near the center.

Talbot concludes that the average stress of all the bars will be equivalent to considering that the bars within a proportionate length b, of the footing width, are equally effective.

He defines this distance as:

$$b = B + 2d + \frac{L - (B + 2d)}{2}$$

This length, therefore, defines the width within which steel for the bending moment should be distributed, and also the available area for calculating the fiber stress in the concrete.

This distance b, in the notation used by Professor Talbot, is designated in this book as M, or the width within which steel for bending moment should be placed.

Area of Steel. The steel required for bending moment within the width b is ascertained by equating the bending and resisting moments and solving for A_s in the usual manner.

A simple expression for steel area true for all sizes of footings for a single cap ratio has been found as

$$A_s = \frac{L^2}{14}$$

and platted as curve A.

This is so easily remembered that it is almost as simple to use it as to refer to the diagrams. The effect of change in size of cap is shown by curve P, which gives the correction to be applied to curve A, for any width of cap.

It is of comparative interest to note that 12″ reinforced footings of same mix and size as the unreinforced footings tested failed in diagonal tension under loading from three to four times as great as those causing rupture in bending in the unreinforced type.

The reinforcing of the concrete in direct tension was sufficient to overcome the first line of weakness, whether due to lack of homogeneity generally or specific weakness at a particular place in an example under test, until failure resulted from secondary stresses in the manner illustrated in Figure 7 as diagonal tension.

Disposition of Tension Steel. Nothing conclusive can be derived from the tests as to the most advantageous position for a given weight of steel.

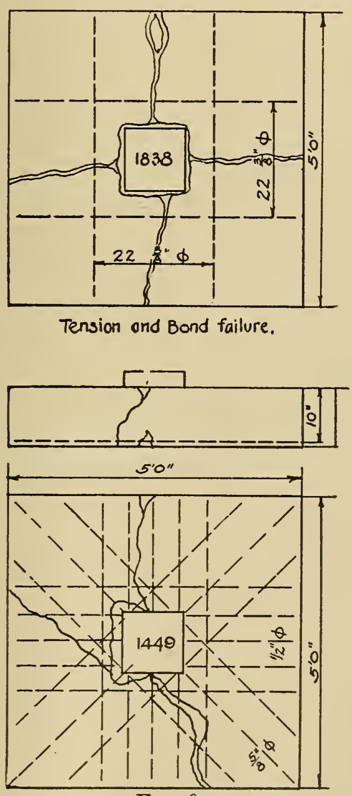


FIG. 9.
ILLUSTRATING FAILURE BY TENSION — CENTER BEAM AND FOUR WAY DISTRIBUTION. *(Talbot.)*

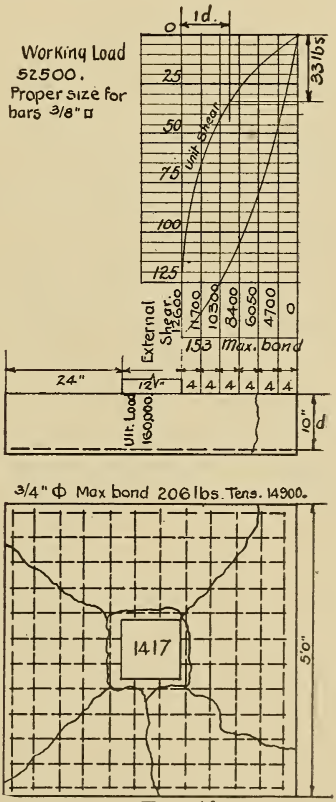


FIG. 10.
TYPICAL CRACKING BY BOND FAILURE. *(Talbot.)*

The results shown by the four way reinforcement do not give any increase of efficiency over two way placement. No. 1564 reinforced to .59% with ½" plain rounds four ways carried 210,000 lbs., while No. 1531 and 1532 with same percentage of same section, two way distribution, carried respectively 280,000 and 252,000 lbs.

The fact that four way reinforcement cracks on the same general lines as in the case with two way disposition might be construed to mean that the diagonal tension bars were not receiving their proportion of the stress, but that the actual distribution is analogous to two way center beams like 1837 and 1838. These, for some reason not yet understood, showed no sign of weakness at the unreinforced corners, although there was a considerable area aggregating 28% of the footing without steel protection. The general tendency, however, of bars placed in bands either centrally or outside is failure in bond.

On the other hand, the general superiority of small bars well distributed through the section is apparent, this result being attributed to better bond conditions.

A reduction in bond resistance was shown when steel was placed within one inch of bottom surface of concrete.

The center of horizontal steel is taken as being 3" above bottom of concrete in these designs, which should be regarded as a minimum to be increased when required by ordinance conditions.

As two way placement has the advantage of relative simplicity, it has been adopted as the method to be used for design.

The steel for bending moment is distributed in width *M*, one bar being placed on line with each end of this distance, and the remaining intermediate bars spaced evenly between.

The steel required for width *O* should be spaced evenly between the end of *M* and the outer face of footing.

Diagonal Tension. If the results of diagonal tension failures, tabulated on page 104 of Bulletin 67, are grouped under similar percentages of reinforcing, average values of vertical shear in an ascending scale are secured as follows:

.003%	.004%	.005%	.006%
120 lbs.	124 lbs.	136 lbs.	160 lbs.

This would indicate that diagonal tension is so inter-related to direct tension and bond that it can not properly be assigned a square inch value as a characteristic of concrete.

In other words, the stress becomes critical after the change of conditions following the destruction of effective tension resistance, when the steel passes its elastic limit, or the critical point of the concrete bond is reached.

Any attempt to measure its unit value will, therefore, be directly affected by the percentage of reinforcement within such limits that other types of weakness do not become critical.

The following data has been selected from page 104 as presenting failures by diagonal tension, most clearly distinguished from other types of weakness.

No.	Concrete.	Depth Over All.	Depth to Steel.	Bars.	Percentage.	Value of J.	Disposition.
1447	1-2½-5	12	10	11 pl. Ø	0.31	.90	6 par. to each s, 5 par. to each d
1448	1-2½-5	12	10	11 pl. Ø	0.31	.90	Do.
1552	1-2½-5	12	10	10 ½ sq. cor.	0.42	.89	6″ C. to C. in 2 way
1808	1-2 -4	12	10	27 ⅜ pl. Ø	0.50	.895	2¼″ C. to C. 2 way
1810	1-2 -4	12	10	33 ⅜ pl. Ø	0.61	.89	1 13/16″ C. to C. 2 way
1435	1-2½-5	11½	10	5 ¾, 2 ½, 2 ⅓ & 2 ¼ cor. sq.	0.61	.87	5″ C. to C. 2 way
1554	1-2½-5	12	10	15 ½ cor. sq.	0.62	.87	4″ C. to C. 2 way

46/60ths of steel considered in figuring percentage of reinforcement, and 46/60 of area of concrete above steel for same purpose.

Calculated Stresses.

No.	Load at Failure	Steel Tension	Vert. Shear at 1d	Bond Stress	First Cracks	Others		Failure
1447	208000	42100	130		144000 center of W. face	160000 N. face center	167 S. face center	Sudden
1448	176000	35600	110		144000 3 faces center			Sudden
1552	236000	45100	149	415	102000 3 faces center			Violent
1808	198000	31600	218	218	120000 N. face center	138000 N.E. & W. center		Diag. Tens.
1810	219000	28700	198	198	2 faces center			Diag. Tens.
1435	208000	22000	134		136000 W. face center	144000 E. face center		Diag. Tens.
1554	288000	37800	185	346	120000 center			Diag. Tens.

No web reinforcement of any kind was present in any footings tested.

All footings five (5) ft. square with a cap 1 ft. square.

In order to relate the stresses calculated from these experiments to working design, the working load, for which the dimensions used are adapted, will be calculated on the assumption that limit shear at 120 lbs. is the governing factor.

Then external shear $V = (4)(12)(10)(7/8)(120) = 50400$.

Column load $(25/24)(50400) = 52500$ lbs.

Soil load $52500/25 = 2100$ lbs. sq. ft.

The reinforcement used bears no direct relation to any design load, as its amount was varied to observe resulting changes, but the dimensions used for all the footings are adapted to working soil reaction of 2100 lbs. per square foot for maximum limit shear of 120 lbs. at the face of cap. The soil pressures created by test loads, and the unit vertical shear at face of cap at the time of first crack and at ultimate loads, have been calculated as follows:

	SOIL LOADS			UNIT SHEAR AT FACE OF CAP		
No.	1st Crack	Ultimate	Design	1st Crack	Ultimate	Working
1447	5700	8300	2100	340	490	120
1448	5700	7000	2100	340	415	120
1552	4000	9500	2100	237	550	120
1808	4800	7900	2100	281	466	120
1810	5500	8800	2100	322	515	120
1435	5400	8300	2100	308	474	120
1554	4800	11500	2100	274	655	120

It is apparent from the above tabulation, that a factor of 2 against first sign of critical weakness in diagonal tension existed in the poorest specimen without special reinforcement.

Compare the bond stress in two typical examples, as, for instance, No. 1552 and No. 1810. Assume bars 56″ long.

No. 1552. Perimeter 2 $\Sigma 0 = 20$

Effective (46/60) (20) = 15.35. Effective length (56 — 12)/2 = 22″.

Then (15.35) (22) = 338 sq. in. bond area available.

Tensile stress @ (45100) (10) (46/60) (.25) = 86600.

Unit bond resistance developed, 86600/338 = 256 lbs.

No. 1810. Perimeter 1.18 $\Sigma 0 = (33)(1.18) = 39$.

Effective (46/60) (39) = 30″.

Then (30) (22) = 660 sq. in. available bond area.

Tensile stress @ (28700) (33) (46/60) (.11) = 80,000.

Unit bond resistance developed, 80,000/660 = 121 lbs.

The results are consistent with the idea that the low diagonal tension failure was associated with a relatively high bond and tensile stress.

The significance of the cracks should also be considered.

The cracks were practically all tension cracks generally located in line with one face of the cap, and the final fracture is typical in plan with those observed in unreinforced footings, representing beam failure in bending. See Fig. 6.

It appears that first tension cracks do not penetrate beyond the rupture plane for diagonal tension, since the central frustrum is intact with cap after failure.

Also the breaking away of the rods from the concrete, outside the frustrum constituting bond failure, finally destroys the tension resistance after diagonal tension cracks are formed, and permits the sudden separation in diagonal tension and the breaking by beam action as in plain concrete.

This indicates that low bond values for working stresses and rein-

forcement against incipient diagonal tension will generally increase the factor of safety of a footing.

Tension cracks weaken bond resistance, and ultimately bond weakness permits final failure by diagonal tension. It is practically impossible to disassociate these factors.

It should be remembered that ultimate failure developed resistance from one and a half to two times greater than that shown at first crack.

Talbot affirms in general that web reinforcement takes no stress until cracking occurs, but as incipient diagonal tension cracks must exist internally long before tension or bond weakness becomes critical, the presence of reinforcement normal to the plane of rupture would evidently assist in preventing failure from this cause.

It appears from the facts noted that while reinforcement for diagonal tension might not be needed if concrete were as reliable as steel, yet it should be used anyway to guard against locally defective construction, peculiar to the material.

Reviewing the results presented by No. 1552:

		Tension	Bond Max.	Average	Shear at face of cap	Shear at 1*d*
Load at first crack....	102000	19500	179	111	237	64
Load at ultimate.....	236000	45100	415	256	550	149
Load at working limits	52500	10200	92.5	57	120	33

A diagram showing the lateral distribution of vertical shear and bond from the face of cap to the outer edge of footing 1817 is shown in Figure 11. The bond curve takes the ordinary parabolic form, while the curve of the shear is an inverted parabola, similar to the bending moment curve.

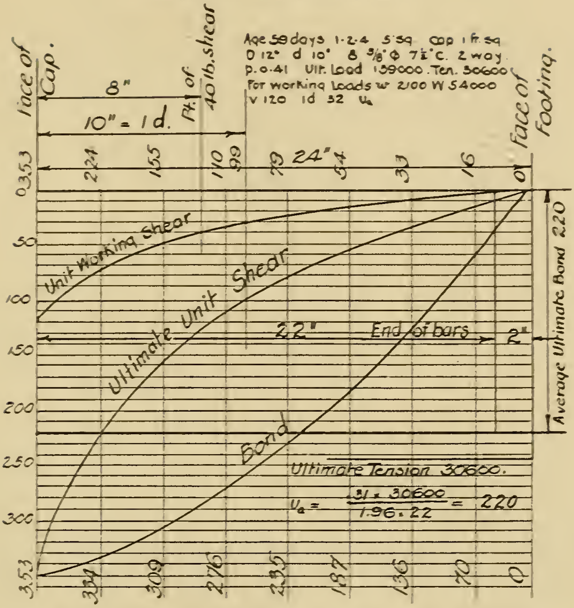


FIG. 11. ILLUSTRATING DISTRIBUTION OF BOND AND VERTICAL SHEAR—FOOTING 1817 *(Talbot)*. FAILURE BY BOND.

Local ordinances fix position at which vertical shear in excess of 40 lbs. allowed in the concrete is to be used as an index for diagonal tension.

This position is usually expressed as 1*d* from face of cap, *jd*, *d*/2, or some other fraction of depth.

In the case of footings under test, the point selected at which to measure vertical shear as an index of diagonal tension is beyond the point of 40 lbs. shear at working limit of of 120 lbs. maximum shear at face of cap. See Fig. 11.

Many designers accept this as warrant for omitting web reinforcement altogether. This is not only contrary to the spirit of the 1913 Joint Committee report allowing 120 lbs. shear at face of cap, when all tension normal to plane of shearing is provided for by reinforcement, but involves a distinctly inadequate factor of safety for a material of known uncertainty of deportment.

Other designers ascertain the unit shear at the point selected, and if in excess of 40 lbs., reinforce for two-thirds of the stress in the steel, but provide nothing, if less than 40 lbs. For 42 lbs. unit shear they

provide steel for 28 lbs., but if the unit shear is 39 lbs. omit reinforcement entirely.

Others again provide steel for two-thirds of the unit stress and presumably, where the point fixed is beyond the point of 40 lbs. shear, assume some related distance to use in calculating the sum of the shear.

The range of application varies from nothing to the provision of absurdly large amounts of web steel.

The greater number of footings are probably constructed for soil pressures ranging from 4000 to 8000 lbs. per square foot, and with ratios between B and L ranging from .25 to .45. Illustrating the range of values of the factors of d/L:

Soil—	4000	5000	6000	7000	8000
K........	.11	.14	.17	.20	.23
B/L......	.25	.30	.35	.40	.45
C........	2.08	1.77	1.48	1.22	1.04

It will be seen that the minimum values of products of these factors would occur with low soil values and high cap ratios. But low soil values mean large footings and relatively low cap ratios. Take an actual case of a hooped column 24″ square to carry 400,000 lbs. at 4000 lbs. soil reaction. This would require a footing 10 ft. square and a cap 3.33 ft. square at most, or

$B/L = 3.33/10 = .333.$
Then $CK = d/L = (1.56)(.11) = 0.17$, or
$d = 0.17\,L.$

It will be seen from examination of the curve of S on diagrams that the distance from the face of cap to point of 40 lbs. shear is as follows:

B/L	.25	.30	.35	.40	.45
Side of square for 40 lbs. shear...	.56 L	.615 L	.67 L	.71 L	.75 L
$(S/L - B/L)/2$..	.155 L	.1575 L	.16 L	.155 L	.15 L

The values in line 2 represent the sides of a square, where 40 lbs. shear exists in footings having B/L as tabulated. Then deducting width of cap and dividing by 2 will give the distance out from face of cap to point of 40 lbs. shear as a fraction of L. It will be seen that within the limits considered values of this distance range from $.15L$ to $.16L$.

As the least probable depth for footings within same limits was found to be $.17L$, it is evident that, with but few exceptions, the point $1d$ from face of cap will be beyond the vertical plane where vertical shear of 40 lbs. exists.

In the case of the footings tested by Prof. Talbot it has already been shown by Figure 11 that only 30 lbs. of shear existed at working limits $1d$ from face of cap.

This may be checked from diagram:

For $B/L = 1/5 = .2$, $C = 2.84$.

For working soil value 2100 lbs., $K = .059$;

$$CK = (2.84)(.059) = .167 = d/L.$$

Then $d = .167L$, or, $(.167)(5')(12'') = 10''$.

For point of 40 lb. shear read across on $B/L = .2$, to curve of S, and find value on Lower Scale of $.475L$.

$(.475 - .2L)/2 = .1375L = (.1375)(60) = 8.25''$.

It will be noticed that the position of the point of 40 lbs. shear at working loads was about one-third the width of the projection of 24″ beyond the face of the cap, and as the effective depth of the footing was 10″, the previous conclusion that the point $1d$ would be beyond the point of 40 lbs. shear is verified, even with such a low soil value as 2100 lbs.

It can also be shown that the use of $\frac{1}{2}d$ is inconsistent, since it must be assumed that the purpose of the ordinance in providing a method for measuring diagonal tension is to require some minimum amount of effective web reinforcement in all cases.

It will be seen again that, since the distance of the point of 40 lbs. shear is from $.15L$ to $.16L$ from the face of cap, $\frac{1}{2}d$ will be at or beyond this point in all footings having a value of d/L of .32 or over.

Examination of the factors of d/L between soil values of 4000 and 8000 lbs. show that a soil value of 8000 lbs. and a B/L of .35 give $d/L = .34$, or $\frac{1}{2}d$ is slightly beyond the point of 40 lbs. shear.

The same result would follow if the soil pressure were 6500 lbs. with a value of K of .184, and a B/L of .30 having a value of $C = 1.77$.

The product of these two factors is $(1.80)(.184) = .33$, and one-half of this amount evidently coincides closely with the point of 40 lbs. shear which has been shown to be $.1575L$ from face of cap.

It may be accepted as true that many Building Departments will continue to permit the omission of web reinforcement under such circumstances until ordinances are changed, but it is well for responsible architects and designers to know that, at best, they are dependent for a somewhat uncertain factor of safety of about 2 upon the mix and placement of their foundations being equal to those secured in laboratory experiments.

Pending the readjustment of official standards, conservative designers will provide what they may regard as a reasonable minimum

regardless of the fact that a slight saving may be made by misusing official sanction.

It should be understood that the steel used is to insure the validity of the factors of safety of about 2 already existing in the concrete as against uncertainties of workmanship and inspection.

If a local pocket of loose aggregate exists in the web by reason of carelessness, the resulting tension crack will induce secondary stresses, which would ordinarily be within the capacity of the concrete to resist, but, because of the local pocketing and incipient diagonal tension cracks, these stresses are transferred to the steel crossing the plane of rupture. It is evident that such a pocket would be localized even in the worst work, and could only be a fraction of total effective resistance afforded by the concrete itself. Using an average value of $B/L = .4$, the sum of the entire vertical shear from the face of the cap to the point of 40 lbs. shear is:

Formula (9).

$$v = (80/2)(.706 + .40)/2)(.155)(144)L^2 = 494L^2.$$

The actual sum is somewhat less than this, because the bounding line of the shear curve is parabolic in form and the assumption of a triangular distribution, substantially correct for small values, would probably involve an over-estimate of 10% with such a large value as 80 lbs.

If this result were applied to a footing 10 feet square with a soil pressure of 6000 lbs. per square foot and a depth of $C K L = (1.22)(.17)(10) = 2.07$ ft., the area of bent steel in each direction would be

$$(0.7)(494L^2)/16000, \text{ or } 2.14 \text{ sq. ins.}$$

It is hardly conceivable that a local irregularity of placement or mix could extend over 10% of the section, because, even assuming the existence of a pocket where the diagonal tension stress is maximum, one cannot practically imagine the reduction of effective section to such an extent. Also, it is a fact that there would be no actual pocket, but only a reduction of the effective resistance of the defective portion of the concrete.

As the steel commences to take stress in proportion to the weakness of the concrete, it seems probable from analogous experience with the deportment of tension steel in the presence of web weakness, that the deficit in strength would be distributed through all the steel crossing the section of weakness. The direct tension steel would be

$$As = (.98)(7.15) = 7 \text{ sq. ins.}$$

The proportion between the tension steel and that for diagonal tension would then be $2.14/7 = 31\%$.

If 10% of the total sum of vertical shear is used as a measure of minimum reinforcement, where ordinance permits the omission of web reinforcement entirely, there would be required each way .214 sq. in. or area equivalent to 3.1% of the tension steel.

It will be found that this is practically equivalent to the result that would follow selecting the place where 60 lbs. of vertical shear exists as the point to measure vertical shear as an index of diagonal tension, as follows:

$$A_{vs} = \frac{\frac{60-40}{2}\ \frac{.706\ \ 6.06}{2}\ (.05)(144)\ L^2}{16000} = .296 \text{ sq.ins.}$$

or 3% of the tension steel.

It will be seen from the form of the last equation that if the value of v were a known constant, fixed by competent authority, as for instance 60 — 40 lbs. as a limit to define the minimum of reinforcement, that the equation would become:

$$A_{vs} = \frac{\frac{(20)}{2}\ (.05)\ (144)\ \frac{(\frac{S}{L}+\frac{S_1}{L}}{2}}{16000} L^2, \text{or}$$

$$A_{vs} = (.0045)\frac{\left(\frac{S}{L}+\frac{S_1}{L}\right)}{2}\ (L^2)$$

or in the above case, (0.7) (.0045) (.706 + .606)/2(100) = .21 sq. in.

This is possible because the value of y in Equation (7) for any given value of v is practically a constant throughout the range of B/L in use and may, therefore, be substituted by its average numerical value, which for 20 lbs. in excess of 40 lbs. allowed in concrete is .05.

It will be seen that the last equation is identical in form with general formula (7).

The values of S/L and S_1/L are obtained by inspection from the diagram for B/L used, and the minimum area of steel varies with L^2, or is as the area of the footing.

Any argument or contention that the point of 60 lbs. shear is a more suitable point than another would be valueless, but no one is apt to contend that less than the amount of steel required by this selection should be used. The limiting extremes have been pointed out and any intermediate point required by local authorities may be used with comparative knowledge of its import.

As will be shown later in the discussion of bond, from fifteen to twenty-five per cent. of the tension steel can be bent up without any significant increase of expense other than for bending labor.

The amount required for diagonal tension on the basis of designing for 20 lbs. of vertical shear in the steel will be from 4% to 5% for stirrups and from 3% to 3½% for bent bars.

It follows, therefore, that for practical designing purposes the intelligent disposition of the material available will probably cover the local requirements anywhere.

If vertical stirrups are used, the continuous type made of ¼″ plain round or smaller sizes insures satisfactory bond conditions without calculations, secures thorough distribution of web resistance and involves practically negligible expense.

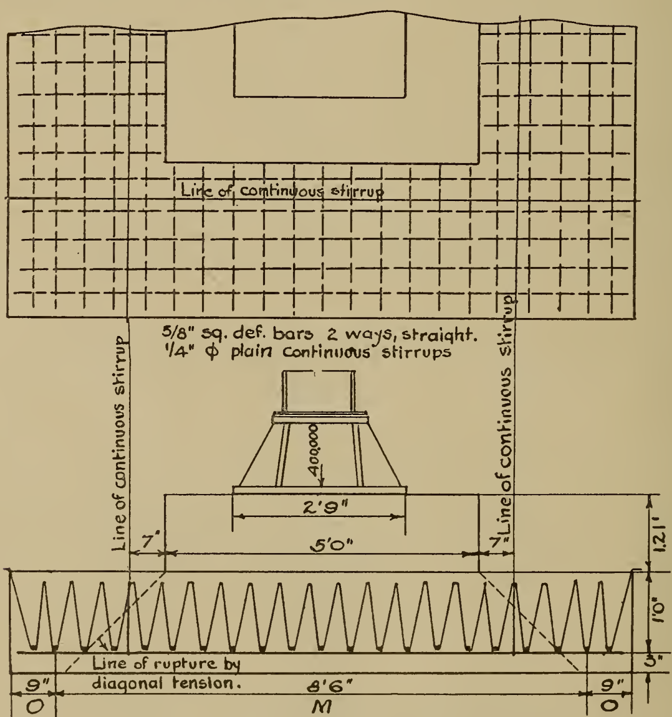


FIG. 12. ILLUSTRATING USE OF CONTINUOUS STIRRUPS FOR SHALLOW FOOTINGS IN PLACE OF BENT BARS.

Suppose that in the footing just used for illustration the tension steel was provided by 18 ⅝ square deformed bars in each direction for width M (See Fig. 12)—

$$M = 4 + 4.14 + \frac{10 - 8.14}{2} = 9.07 \text{ ft.}$$

Then width O requires

$$\left(\tfrac{3}{4}\right)\frac{\left(\frac{10-9.07}{2}\right)}{10}(18) = .62 \text{ bars}$$

As each bar is .39 sq. in. area for $O = (.39)(.62) = .24$ requiring 1 ½" square bar.

There are then twenty bars to be placed in the width of the footing.

Absolutely accurate spacing laterally and vertically may be secured by using 4 continuous stirrups with as many double legs as there are tension bars.

The tension bars are suspended in their correct relative position with no ties required between the stirrups and the tension steel.

The amount of steel required will be roughly—

(20) (2) (2′ 0″) = 80 ft., or
80 ft. @ .167 lbs. = 13.35 lbs.

13.35 lbs. @ 2¢ base	26.7¢
13.35 lbs. Standard extra @ .50 . . .	6.7¢
Bending	4.0¢
Placing	2.0¢
Total cost per stirrup	39.4¢

The figures for bending and placing are made up from actual cost figures. The wire is wound around pegs on the shop table in form of a figure 8, is sent to the job in that shape, and pulled out to space and length when placed. Seventeen double legs are effective in width M as diagonal tension reinforcement, or

$$(34)(.049) = 1.67 \text{ sq. ins.} = 23.7\%$$

of area of tension steel.

At first glance this arrangement seems objectionable, but study of its advantages will show many strong points. The ends of the continuous stirrups are extended up and hooked over the edge of the forms, and the stirrup is supported at intermediate points on fragments of brick, making a strong, reliable support for the tension steel.

When considered as a percentage of the cost of tension steel, cost will be seen to be a negligible factor of expense. The cost of tension steel would be:

36 bars ⅝ sq. @ 1.33 × 9.67	465 lbs.
4 bars ½ sq. @ .85 × 9.67	33
	498 lbs.

498 lbs. @ 2¢ base	$ 9.96
Placing @ ½¢	2.49
Total cost	$12.45

Cost of stirrups, 1.57/14.02 = 11% of total steel cost.

In the same way it might be shown that the stirrups represent less than 3% of the cost of the entire footing. This might be slightly reduced by using steel proportioned to the actual stress, but as it represents the practical minimum for efficient handling and is evidently much more convenient than a multiplicity of separate stirrups, it makes a convenient element in standardized designing.

It further illustrates the fact that the design will be independent of the exact point selected at which to measure diagonal tension.

The position of web reinforcement will be considered in connection with the subject of bent bars, but the general conclusions may be summarized as follows:

Minimum amount of reinforcement will be determined by position of measurement fixed by ordinance or interpreting authority.

Maximum amount will be fixed by the designer through the use of—

1st. A percentage of tension steel bent up.

2d. Continuous vertical stirrups.

3d. A combination of 1 and 2.

The amount of diagonal tension steel required may vary from nothing to as much as 40% of the tension steel.

From 15% to 30% of tension steel may be bent up without affecting working bond limits for remaining steel, with negligible expense.

Regardless of requirements, not less than 10% of tension steel should be bent up, or equivalent resistance in the form of vertical stirrups provided.

Continuous stirrups representing from 20% to 50% of tension steel area involve negligible expense, and for important work may be combined with bent steel.

Conclusions reached by Professor Talbot on beam experiments leading to the recommendation that both bent bars and stirrups are desirable, seem applicable to the conditions presented in footing design, both from the standpoint of assured stability and of negligible cost.

Disposition of Web Steel. While the vertical shear is maximum on the line of the edge of cap, it has been shown that rupture by diagonal tension occurs before punching shear becomes critical; also, that the tension cracks preceding diagonal tension failure do not extend across the plane of rupture by diagonal tension, and that the frustrum of a pyramid produced by the planes of rupture by diagonal tension is integrally intact with the cap after rupture.

It follows that no diagonal steel will be needed across the vertical plane in line with the cap, but that as the decrease of bending moment permits tension steel may be bent up at an angle of 45°. Its ideal position from the consideration of symmetry of section would be such, that it would cross the center of the plane of rupture, which would be somewhat below the geometrical center of the section above the steel.

The clearance of bent up steel below the upper surface of concrete will be from 2 to 3 inches, and the point of bending will ordinarily be from 3″ to 8″ out from the face of the cap.

This means that its intersection with the plane of rupture will be somewhat below the point indicated, but as the diagonal tension cracks start at the bottom of the footing, a compensating advantage is gained through the steel crossing the rupture plane lower down, and also an increase of available anchorage.

When excess steel beyond requirements is being considered, any necessary addition to grip length may be obtained by decreasing the angle of 45° instead of making a second bend above.

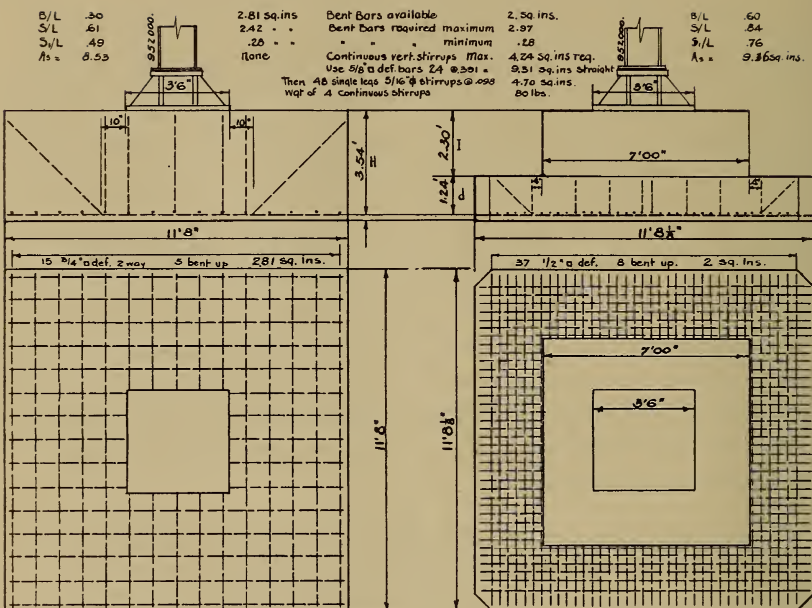
The area of steel required will be increased as the sine of the angle between a vertical from the point of bending and the inclination of the bar.

Where in shallow footings the point of bending is so close to the plane of rupture as not to afford a satisfactory position, continuous stirrups may also be employed.

When used these should be located at the point where a vertical line intersects the plane of rupture half the length of the stirrup above the tension steel. This position insures an equality of anchorage above and below to the maximum possible extent, a consideration of much more importance than its location in the center of gravity of the shear area used as an index.

The location of this index merely establishes the magnitude of the assumed stress, and does not determine the most advantageous position for the steel to resist it.

Calling the clearance for the steel below the upper surface of the concrete C, the advantageous position of continuous stirrups will be $(d - c)/2$ beyond the face of cap.



B/L	.30	.35	.40	.45	.50	.55	.60	.63	.67	.73	.76
H	3.54	3.54	3.54	3.54	3.54	3.54	3.54	3.54	3.54	3.54	3.54
I	.00	.58	1.10	1.46	1.78	2.06	2.30	2.42	2.58	2.78	2.91
d	3.54	2.98	2.44	2.08	1.76	1.48	1.24	1.12	.96	.78	.63
Cap size	0.00	4.08'	4.65	5.23	5.81	6.40	6.99	7.33	7.80	8.50	8.83
Cap Concrete	.00	10.	24.	40.	60.	85.	112.	131.0	157.	202	229.
Base Concrete	482.	403.	332.	283.	240.	201.	169.	152.	131.	103.	85.
Total Concrete	482.	413.	356.	323.	300.	286.	281.	263.	288.	305.	314.
Sq. Ins. Steel	8.53	9.16	9.60	9.75	9.73	9.62	9.18	8.86	8.38	7.41	6.81
Wgt. Steel.	657	708	741	753	753	742	708	684	646	572	526
Cost of Steel	$15.10	16.25	17.06	17.30	17.30	17.07	16.25	15.70	14.85	13.15	12.10
Cost Concrete	120.50	103.25	89.00	80.75	75.00	71.50	70.25	70.75	72.00	76.25	78.50
Total Cost.	$135.60	119.50	106.06	98.05	92.30	88.57	86.50	86.48	86.85	89.40	90.60
Cap Depth / Cap projection	0/0	2/1	1.9/1	5/3	3/2	1.46/1	4/3	5/4	6/5	11/10	13/12

FIG. 13. ILLUSTRATING ECONOMIC DESIGN.

Bond Stress. The usual method of calculating bond is to consider satisfactory, if the product of the bar perimeter, the length of bar beyond the point of maximum tensile stress, and the allowable working bond stress is equal to, or greater than, the product of the bar area and its allowable tensile stress.

Working bond stress is, therefore, the average stress to which the concrete along a reinforcing bar is subjected per square inch of contact.

This is also the meaning attached to the term in building ordinances.

A method of selecting a proper size of bars for known conditions to develop exactly allowable limit bond stress has been devised that permits this result to be attained without calculations by a mere inspection of the bond curves. Each curve is designated on the diagram by the size of the bar above and its perimeter below the line.

The curves are points of equal bond stress for bars of given size. Since the value of the resistance for a given bar is as the surface area of a unit length, the variation in resistance between bars of different size will be directly as their perimeters. But the variation of tensile strength is directly as the areas, or as

$$\left(\frac{O}{4}\right)^2 = \frac{O^2}{16}$$

It is then evident that if a bar, having a perimeter of 4, furnishing (under the conditions of its placement) equal resistance to tension and bond, is divided into four bars under the same conditions, that while the combined tension resistance of the four bars remains the same, the bond resistance is doubled, or is inversely as the perimeters. Illustrating:

Perimeter of given bar . . . . 4.

Area of given bar

$$\frac{O^2}{16} = \frac{4^2}{16} = 1$$

Area of each of 4 bars 0.25

Perimeter of divided bars . . . . 2.

Sum of divided perimeters . . . . 8.

Ratio of perimeters 4/8

Since there are four bars, it is then evident that two of them provide resistance to bond stress equivalent to the tensional value of all. As this merely represents the law of arithmetical relation between area and perimeter, it is true for all perimeters, and a general rule, availed of in the use of these diagrams, may be stated, viz.: The amount of steel of a given size, required to develop bond resistance equivalent to the tension value of all the tension steel at the critical point, may be found by multiplying the area of tension steel required for maximum bending moment by a fraction, whose denominator is the perimeter of a bar that would provide equality of resistance to working stresses in tension and bond, and whose numerator is the perimeter of the bar to be used. The difference between the amount so determined and the amount required for maximum bending moment represents the area that may be bent up, whenever the decrease of bending moment permits.

In using this simple fact with the diagrams, it will be seen that, while the distance between the curves measured on vertical lines varies, the value of the separation is uniformly one-half. That is, the lines are contours of equal value one-half unit apart.

If absolutely exact determination of the values is desired, the co-ordinate spaces, between the curves, may be counted and the position of the intersection of B/L and L precisely located on this scale, but the eye will easily assign the position of this point a proportional value of sufficient accuracy for the purpose in hand.

Illustrating by the use of data in Example 1:

The intersection of B/L and L is about 1/5 (actually 1.8/9) below the ¾″ curve, which has a perimeter value of 3. Then this fraction represents 1/5 of one-half (the increase of perimeter value in the distance to the next curve below), or

$$(1/5)(\tfrac{1}{2}) = .1$$

Then 3.1 represents the perimeter of a bar that would furnish exactly equal resistance in tension and bond, under the stated conditions of placement.

Checking this conclusion:

The projection of footing, $(10 - 4.5)/2 = 33$ inches	
Clearance for steel at face of footing . .	2
Length of bar available for bond resistance	31 inches

Then bond resistance = (3.1) (31) (100) = 9610 lbs.

Area of bar

$$\frac{3.1^2}{4} = \frac{9.61}{16}$$

As tension in steel is 16000 lbs., the tension value of this area is—

$$(16000)(9.61)/16 = 9610 \text{ lbs.}$$

Now, as some steel is to be bent up, and as excess will be beneficial so long as the area is sufficient for the estimated diagonal tension without diminishing the allowable resistance in bond and tension, ⅝ bars having a perimeter value of 2½ will be used. Then the number of bars required in tension will be—

7.13/.391	= 18.30
Number of bars required in bond, (2.5/3.1) (18.3)	= 14.75
Number of bars available for bending up . .	3.55

Checking these conclusions:

Total tension, (18.3) (.391) (16000) = 114000.

Total bond, (14.75) (2.5) (31) (100) = 114500.

The inaccuracy indicated in the check of about four-tenths of one per cent. is, of course, negligible and is probably due to error of position in platting of curves.

The simplicity and precision of the method used contrasts favorably with the usual custom of guessing at the size of bars, and then accepting the result as O. K. because needless excess strength is found to be available.

The tension value of any bar of known perimeter can be figured for working stresses of 100 lbs. and 16000 lbs. respectively in bond and tension without determining the area, by the formula:

Tension value, $(1000)(O^2)$ for square bars.

Tension value, $(1273)(O^2)$ for round bars.

Also the greater relative efficiency of square bars as regards bond may be illustrated as follows:

	Round	Equal Diam. Square	Equal Perim. Square	Equal Area Square
Diameter	1.00	1.00	.7854	.886
Perimeter	3.1416	4.00	3.1416	3.544
Area	0.7854	1.00	.6169	.7854

It thus appears that, for bars of same area, square bars give greater bond resistance than round ones.

Also, it may be stated that while many bond failures with plain bars were noted in the footings tested, only one failure occurred with deformed bars, and this was at the relatively high unit stress of 596 lbs. per square inch.

One of the normal failures with plain bars was at a unit stress as low as 223 lbs. The relations will be seen in tabulated form more readily.

	Working Stresses: Average	Maximum	Ultimate Failure	Factor of Safety
Plain bars...........	80	130	223	1.72
Deformed bars.......	100	163	596	3.66

From "pulling out" tests at the University of Illinois, the following was obtained:

	Plain Lbs.	Johnson Lbs.
Maximum	360	639
Minimum...........	174	298
Average	281	484

It should be emphasized that full size working tests do not usually realize the unit values derived from "pulling out" and "pushing out" tests. Such tests, however, appear to establish a ratio between plain and deformed sections that is reasonably dependable, viz., that the minimum values for deformed bars are relatively much higher than the low records for plain bars. It would appear from the observations here considered that the working stress for plain bars is too high for reasonable safety.

Anyway it will be seen that for the working values used in the diagrams it is distinctly advantageous to use square deformed bars.

It will also be useful to remember that other tests indicate that bond resistance is increased by the presence of vertical stirrups.

The bond curves have been platted for this condition of maximum efficiency, viz., square deformed bars @ 100 lbs. working stress, to make the best way the easiest way.

Other values of working stress, either larger or smaller, may be integrated into the results by multiplying the perimeter, affording equal resistance in tension and bond, by a fraction whose numerator is the stress to be used, and whose denominator is 100, the stress for which the curves are figured.

The importance of bond has been appreciated by very few designers. The experiments of Prof. Talbot emphasize its importance and cause him to express a caution as to the consideration to be given it.

It is analogous to providing sufficient rivets in a plate girder to develop the full working strength of the material required in the flanges and web to resist the external load stresses, and to such distribution of these that the greatest localized intensity of stress will not exceed the safe capacity of one rivet.

In the same way, the measure of maximum tensile stress that can be developed in the reinforcing steel will be the safe capacity of the concrete in shear at the critical point to take off the decrement of stress, this resistance being measured as bond stress.

It, therefore, becomes important to understand the distribution of bond stress, its maximum intensity, and relation to average bond stress as used in the meaning assigned to it in building ordinances.

Maximum intensity and distribution may be determined by Equation (15)—

$$U = \frac{V}{m_o j d}$$

as shown in Figure 11, which represents the bond curve for one of the Talbot footings.

The maximum bond ordinate evidently occurs where the bending moment is maximum, and, since the entire external shear is used, assumes that the bars extend to the face of the footing.

The bounding line of the bond ordinates is a parabola, and the value of its mean or average ordinate is, therefore, 2/3 of the maximum intensity.

If this were a known constant relation, Formula (15) would be a useful one for design, as factors of safety should be definitely related to the maximum intensity of stress rather than to average values.

Actual examples show that the relation is not uniformly constant.

Until this relation is rationalized and ordinances are changed accordingly, it is necessary to continue the present method, but it is evidently desirable to investigate the relation as its exists in typical instances of design and in experiment.

The following tabulations are from diagram calculations:

Soil	B/L	d	As	Tension Capac. @ 16,000 lbs.	Bars	Size	Available for Bond	Effective Bond Length	Bond Resist. @ 100 lbs.
7000	.3	3.54	6.25	100,000	11.15	¾ sq.	8.37	40	100,400
7000	.6	1.24	6.72	107,500	48.	⅜ sq.	33.	22	108,900

Then for

$$\text{No.1} \quad U = \frac{(.228)\ (700,000)}{(8.37)(3)(7/8)(3.54)(12)} = 171.5$$

$$\text{No.2} \quad \frac{(.16)\ (700,000)}{(33)(1.5)(7/8)(1.24)(12)} = 174.$$

In these cases the average bond is figured for bars having a clearance of 2″ from face of concrete, while the maximum bond uses the entire external shear so that the results are not strictly comparable for other conditions than those stated.

For use with the diagrams, however, it is evident that the use of 100 lbs. working bond stress involves maximum stress at critical points of about 175 lbs. per square inch.

The following is from Bulletin 67, University of Illinois:

Soil	B/L	d	As	Tension Capac. @ 16,400 lbs.	Bars	Size	Available for Bond	Effective Bond Length	Bond Resist. @ 146 lbs.
1414	.2	0.83	4.06	66,300	9.17	¾ Ø	9.17	21	66,300
				@ 34,800 lbs.					@ 155
1806	.2	0.83	1.86	64,500	16.85	⅜	16.85	21	64,500

Then for No. 1418, $u = 226$. Ratio $226/146 = 1.55$.

No. 1806, $u = 241$. Ratio $241/155 = 1.56$.

If all the instances given were figured on the assumption that the bars extend to the face of the footing, the results would be:

No. 1.	$(40/42)(100) = 95.5$	Then	$171.5/95.5 = 1.80$
2.	$(22/24)(100) = 91.6$		$174/91.6 = 1.92$
1418.	$(21/24)(146) = 128.$		$226/128 = 1.765$
1806.	$(21/24)(155) = 136.$		$241/136 = 1.77$

The low values of bond failure given on page 96 of Bulletin 67 are:

No. 1417. Eccentric loading.............. 206 lbs.
1451. Sloped footing................. 131
1833. Outside bands................. 152
1834. Outside bands................. 206
1837. Center bands.................. 198

As the loading of 1417 was not uniform, the results are not comparably determinable. The results in 1451 are clearly due to increase of bond stress, because of change of value of *jd,* through sloping.

The remaining footings apparently give results characteristic of the disposition of the steel, indicating inherent inferiority.

Taking the next group of low values:

No. 1418. Load eccentric................. 226 lbs.
1541. Lack of concrete below bars..... 231
1813. 223
1806. 241

No. 1418 and 1541 may be eliminated for assignable causes, but the remaining two are apparently normal as far as the record discloses the pertinent facts.

The comparable facts are:

No.	Description	p	Load at Failure	Tension	Max. Bond at Failure
1806	22 ⅜ pl. ∅	0.41	179,000	34,800	241 lbs.
1813	12 ½ pl. ∅	0.39	121,000	24,300	223

Other dimensional facts being identical, the advantage of larger load capacity for small bars is apparent.

The fact that only one failure occurred in bond with deformed bars has already been referred to. The data is as follows:

No.	Description	p	Load at Failure	Tension	Max. Bond at Failure
1844	8 ⅝ cor. Ø	0.41	269,000	52,000	596 lbs.

It is obvious that square deformed bars furnish more security in bond than equal areas of any other type.

The distribution of bond in the case of No. 1451 is shown in Figure 14.

This is reported to have failed in bond at the low value of 131 lbs. per square inch.

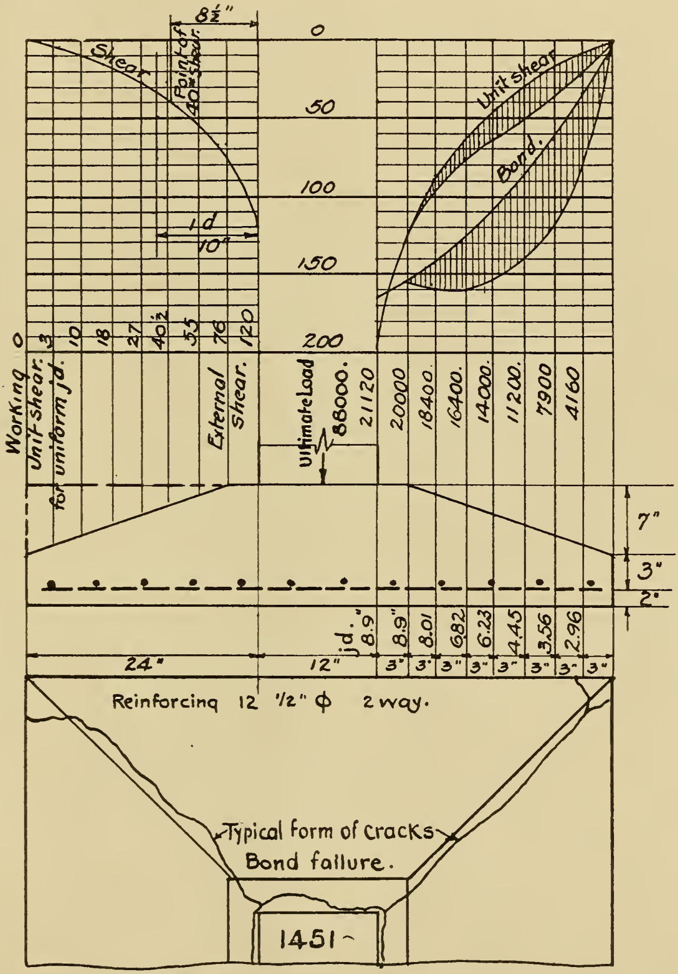


FIG. 14. ILLUSTRATING INCREASE OF BOND STRESS CAUSED BY SLOPING FOOTING. FAILURE BY BOND. *(Talbot.)*

The excess bond caused by sloping the footing is also shown by platting the bond line which would exist without sloping, and hatching the space between the two lines.

The results show why footings should not be sloped, unless the allowable unit bond stress is reduced proportionately to the reduction of jd.

As the figures for maximum bond and shear at $1d$ do not agree with those reported by Talbot, the calculations will be checked.

No. 1451. $p = 0.39$. $j = 0.89$. $d = 0.83$.

Load failure $= 88{,}000$.

Soil pressure $= 88000/25 = 3520$ lbs.

$B/L = 1/5 = .2 - B/L$ at $1d$ from cap $2.67/5 = .534$

For B/L .534 $C\,K\,L = (.78)\,(.10)\,(5) = .39$

jd at $1d$ from cap $= (.89)\,(7.67) = 6.82$ inches

Shear. Then $v = (.39/.83)\,(120)\,(.875/.89)\,(8.9/6.82) = 72.4$ lbs.

Talbot determination 59.0 lbs.

Recheck:

External shear V at $1d$ from cap $= V^{\Delta}W$, or

for B/L $.534 = .18 \times 88000 = 15840$.

Shear. Then $v = 15840/(2.67)\,(12)\,(6.82) = 72.5$ O. K.

Steel Stress. For $B/L = .2$, $C\,K\,L = (2.84)\,(.10)\,(5) = 1.42$

Steel required, $(.675)\,(1.8) = 1.21$ sq. ins.

Steel used, 12 ½″ Ø @ $1.96 = 2.35$ sq. ins.

Effective, $(2.35)\,(46/60) = 1.80$

Then $f_s = (1.40/.833)\,(16000)\,(1.21/1.80)\,(.875/89) = 17800$

Talbot determination 17800

Maximum Bond. $$U = \frac{(.24)\,(88000)}{(12)\,(46/60)\,(1.57)\,(8.9)} = 164 \text{ lbs.} \quad \text{O. K.}$$

Talbot determination 131 lbs.

Staggered Bars. Designers frequently stagger the lengths of their tension steel as the decrease of bending moment permits.

This should always involve an investigation of the actual bond stress available, as this is usually deficient under such circumstances.

Conclusions. The same principles of design may be applied to combined cantilever and raft foundations for depth and steel determination.

It will also be seen that diagrams of the same type for live floor loads from zero to 1000 lbs. may be used to design column caps and flat slabs of the so-called girderless floors.